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COEFFICIENT ESTIMATES FOR SOME CLASSES OF p-VALENT FUNCTIONS

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ABSTRACT. Let A_p, where p is a positive integer, denote the class of functions $f(z) = z^p + \sum_{n=p+1} a_n z^n$ which are analytic in $U = \{z: |z| < 1\}$.

For $0 < \lambda \leq 1$, $|\alpha| < \frac{\pi}{2}$, $0 \leq \beta < p$, let $F_\lambda(\alpha,\beta,p)$ denote the class of functions $f(z) \in A_p$ which satisfy the condition

$$\left|\frac{H(f(z))-1}{H(f(z))+1}\right| < \lambda \quad \text{for} \quad z \in U ,$$

where

$$H(f(z)) = \frac{e^{\frac{i\alpha z f'(z)}{f(z)}} - \beta\cos\alpha - ip\sin\alpha}{(p-\beta)\cos\alpha} .$$

Also let $C_\lambda(b,p)$, where p is a positive integer, $0 < \lambda < 1$, and $b \neq 0$ is any complex number, denote the class of functions $g(z) \in A_p$ which satisfy the condition

$$\left|\frac{H(g(z))-1}{H(g(z))+1}\right| < \lambda \quad \text{for} \quad z \in U, \text{ where}$$

$$H(g(z)) = 1 + \frac{1}{pb}\left(1 + \frac{zg''(z)}{g'(z)} - p\right).$$

In this paper we obtain sharp coefficient estimates for the above mentioned classes.

KEY WORDS AND PHRASES. p-valent, starlike, convex, spirallike functions.
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1. INTRODUCTION.

Let A_p, where p is a positive integer, denote the class of functions $f(z) = z^p + \sum_{n=p+1}^{\infty} a_n z^n$ which are analytic in $U = \{z: |z| < 1\}$. We use Ω_λ, $0 < \lambda \leq 1$, to denote the class of analytic functions $w(z)$ in U satisfying the conditions $w(0) = 0$ and $|w(z)| < \lambda$, $0 < \lambda \leq 1$.

Padmanabhan introduced the class of starlike functions of bounded order λ, $0 < \lambda \leq 1$, defined as follows [11]:

DEFINITION 1. A function $f \varepsilon A_1$ and satisfying

$$\left|\frac{\frac{zf'(z)}{f(z)} - 1}{\frac{zf'(z)}{f(z)} + 1}\right| < \lambda \tag{1.1}$$

for a given λ, $0 < \lambda \leq 1$, $|z| < 1$ is said to be starlike of bounded order λ in $|z| < 1$ and this class is denoted $S(\lambda)$, the class of all such functions for a given λ.

Let $F(\alpha,\beta,p)$ $(|\alpha| < \frac{\pi}{2}, 0 \leq \beta < p)$ denote the class of functions $f(z) \varepsilon A_p$ and for which there exists a $\rho = \rho(f)$ such that

$$\text{Re } \{e^{i\alpha} \frac{zf'(z)}{f(z)}\} > \beta \cos\alpha \tag{1.2}$$

and

$$\int_0^{2\pi} \text{Re } \{\frac{zf'(z)}{f(z)}\} d\Theta = 2\pi p \quad \text{for} \quad z = re^{i\Theta}, \rho < r < 1 . \tag{1.3}$$

Functions in $F(\alpha,\beta,p)$ are called p-valent α-spirallike functions of order β. The class $F(\alpha,\beta,p)$ was introduced by Patil and Thakare [12].

In this paper we use a method of Clunie [3] to obtain sharp bounds for the coefficients of functions F_λ (α,β,p) and C_λ (b,p) , where p is a positive integer, $0 < \lambda \leq 1$, $|\alpha| < \frac{\pi}{2}$, $0 \leq \beta < p$, and b is any complex number, where F_λ (α,β,p) and C_λ (b,p) are defined as follows:

DEFINITION 2. For $0 < \lambda \leq 1$, $|\alpha| < \frac{\pi}{2}$, and $0 \leq \beta < p$, let F_λ (α,β,p) denote the class of functions $f(z) \varepsilon A_p$ which satisfy the condition

$$\left|\frac{H(f(z))-1}{H(f(z))+1}\right| < \lambda \tag{1.4}$$

for $z \varepsilon U$, where

$$H(f(z)) = \frac{e^{i\alpha} \frac{zf'(z)}{f(z)} - \beta \cos\alpha - ip \sin\alpha}{(p-\beta)\cos\alpha} . \tag{1.5}$$

DEFINITION 3. For p is a positive integer, $0 < \lambda \leq 1$, and $b \neq 0$ is any complex number, let $C_\lambda(p,b)$ denote the class of functions $g(z) \varepsilon A_p$ which satisfy the condition

$$\left|\frac{H(g(z))-1}{H(g(z))+1}\right| < \lambda \tag{1.6}$$

for $z \varepsilon U$,

where
$$H(g(z)) = 1 + \frac{1}{pb} \left(1 + \frac{zg''(z)}{g'(z)} - p\right). \tag{1.7}$$

We note that by giving specific values to λ, α, β, p and b, we obtain the following important subclasses studied by various authors in earlier papers:

(1) $F_1(0,0,1) = S^*$ and $C_1(1,1) = C$, are respectively the well-known classes of starlike functions and convex functions, $F_1(0,\beta,1) = S_\beta$ and $C_1(1-\beta,1) = C_\beta$, $0 \leq \beta < 1$, are respectively the classes of starlike functions of order β and convex functions of order β introduced by Robertson [14], $F_\lambda(0,0,1) = S(\lambda)$ and $C_\lambda(1,1) = C(\lambda)$, is the class of functions g for which $zg'(z) \varepsilon S(\lambda)$.

(2) $F_1(\alpha,0,1) = S^{\alpha}$ and $C_1(\cos\alpha\, e^{-i\alpha},1) = C^{\alpha}$, $|\alpha| < \frac{\pi}{2}$, are respectively the class of α-spirallike functions introduced by Špaček [18] and the class of functions g for which $zg'(z)$ is α-spirallike introduced by Robertson [15], $F_1(\alpha,\beta,1) = S^{\alpha}_{\beta}$ and $C_1[(1-\beta)\cos\alpha\, e^{-i\alpha},1] = C^{\alpha}_{\beta}$, $|\alpha| < \frac{\pi}{2}$, $0 < \beta \leq 1$, are respectively the class of α-spirallike functions of order β introduced by Libera [8] and the class of functions g for which $zg'(z)$ is α-spirallike of order β by Chichra [2] and Sizuk [17].

(3) $C_1(b,1) = C(b)$ is the class of functions $g \in A_1$ satisfying

$$\mathrm{Re}\{1 + \frac{1}{b}\frac{zg''(z)}{g'(z)}\} > 0$$

introduced by Wiatrowski [19] and studied by [9] and [10].

(4) $F_1(0,0,p) = S(p)$, $C_1(1,p) = C(p)$, $F_1(0,\beta,p) = S\ \beta(p)$ and $C_1[(1-\frac{\beta}{p}), p] = C_{\beta}(p)$, $0 \leq \beta < p$, are respectively the classes of p-valent starlike functions, p-valent convex functions, p-valent starlike functions of. order β and p-valent convex functions of order β considered by Goodman [6] and the class $S_{\beta}(p)$ investigated by Goluzina [5].

(5) $F_1(\alpha,0,p) = S^{\alpha}(p)$ and $C_1(\cos\alpha e^{-i\alpha},p)$, $|\alpha| < \frac{\pi}{2}$, are respectively the class of p-valent α-spirallike functions and the class of p-valent functions $g \in A_p$ satisfying

$$\mathrm{Re}\ e^{i\alpha}(1 + \frac{zg''(z)}{g'(z)}) > 0, \quad z \in U$$

i.e., the class of p-valent functions g for which $\frac{zg'(z)}{p}$ is p-valent α-spirallike.

(6) $F_1(\alpha,\beta,p) = F(\alpha,\beta,p)$ and $C_1[(1 - \frac{\beta}{p})\cos\alpha\ e^{-i\alpha}, p]$, $|\alpha| < \frac{\pi}{2}$, $0 \leq \beta < p$, is the class of p-valent functions g for which $\frac{zg'(z)}{p}$ is p-valent α-spirallike of order β.

(7) $C_1(b,p)$, is the class of functions $g \in A_p$ satisfying

$$\mathrm{Re}\ \{p + \frac{1}{b}(1 + \frac{zg''(z)}{g'(z)} - p)\} > 0, \ z \in U,$$

the class $C(b,p)$ was introduced by the author [1].

(8) $F_{\lambda}(\alpha,\beta,1) = F_{\lambda}(\alpha,\beta)$, is the class of functions investigated by Gopalakrishna and Umarani [7].

(9) $C_1[(1 - \frac{\beta}{p})\cos\alpha\ e^{i\alpha}, p]$, $|\alpha| < \frac{\pi}{2}$, $0 \leq \beta < p$, is the class of p-valent functions $g(z)$ for which $\frac{zg'(z)}{p} \in F_{\lambda}(\alpha,\beta,p)$.

We state the following lemma that is needed in our investigation.

LEMMA 1[11]. Let $f(z)$ be analytic for $|z| < 1$ and let $f(0) = 0$. Then $f(z) \in S(\lambda)$ if and only if

$$f(z) = z \exp\left[-2 \int_0^z \frac{\phi(t)}{1 + t\phi(t)}\, dt\right],$$

where $\phi(z)$ is analytic and satisfies $|\phi(z)| \leq \lambda$, $0 < \lambda \leq 1$, for $|z| < 1$.

In the rest of the paper we always assume that p is a positive integer, $0 < \lambda \le 1$, $|\alpha| < \frac{\pi}{2}$, $0 \le \beta < p$, and $b \ne 0$ is any complex number.

2. REPRESENTATION FORMULAS FOR THE CLASS $F_\lambda(\alpha,\beta,p)$.

LEMMA 2. $f(z) \in F_\lambda(\alpha,\beta,p)$ if and only if for $z \in U$

$$e^{i\alpha}\frac{zf'(z)}{f(z)} = \cos\alpha\,\{\frac{p-(p-2\beta)w(z)}{1+w(z)}\} + ip\sin\alpha, \tag{2.1}$$

$w \in \Omega_\lambda$.

PROOF. If $f(z)$ is given by (2.1), then

$$H(f(z)) = \frac{e^{i\alpha}\frac{zf'(z)}{f(z)} - \beta\cos\alpha - ip\sin\alpha}{(p-\beta)\cos\alpha} = \frac{1-w(z)}{1+w(z)}$$

so that $\dfrac{H(f(z))-1}{H(f(z))+1} = -w(z)$

and so (1.4) holds. Thus $f(z) \in F_\lambda(\alpha,\beta,p)$.

Conversely, if $f(z) \in F_\lambda(\alpha,\beta,p)$, then (1.4) holds.

Defining $w(z) = \dfrac{1-H(f(z))}{1+H(f(z))}$ we obtain (2.1) and the proof is complete .

LEMMA 3. $f(z) \in F_\lambda(\alpha,\beta,p)$ if and only if

$$f(z) = z^p\left[\frac{f_1(z)}{z}\right]^p \tag{2.2}$$

for some $f_1 \in F_\lambda(\alpha,\frac{\beta}{p},1)$.

PROOF. Let $f(z) = z^p\,[\frac{f_1(z)}{z}]^p$ for $f_1(z) = z + \sum_{n=2}^{\infty} c_n z^n \in F_\lambda(\alpha,\frac{\beta}{p},1)$, $z \in U$.

By direct computation, we obtain

$$\frac{e^{i\alpha}\frac{zf'(z)}{f(z)} - \beta\cos\alpha - ip\sin\alpha}{(p-\beta)\cos\alpha} = \frac{e^{i\alpha}\frac{zf_1'(z)}{f_1(z)} - \frac{\beta}{p}\cos\alpha - i\sin\alpha}{(1-\frac{\beta}{p})\cos\alpha}$$

and the result follows from (1.4).

In a similar way we can prove the following lemma :

LEMMA 4. $f(z) \in F_\lambda(\alpha,\beta,p)$ if and only if

$$f(z) = z^p\left[\frac{f_2(z)}{z}\right]^{(p-\beta)\cos\alpha\, e^{-i\alpha}} \tag{2.3}$$

for some $f_2 \in S(\lambda)$.

An immediate consequence of lemmas 1 and 4 is

THEOREM 1. $f(z) \in F_\lambda(\alpha,\beta,p)$ if and only if

$$f(z) = z^p\exp[-2(p-\beta)\cos\alpha e^{-i\alpha}\int_0^z \frac{\phi(t)}{1+t\phi(t)}\,dt] \tag{2.4}$$

where $\phi(z)$ is analytic and satisfies $|\phi(z)| \le \lambda$, $0 < \lambda \le 1$, for $|z| < 1$.

3. COEFFICIENT ESTIMATES FOR THE CLASS $F_\lambda(\alpha,\beta,p)$.

LEMMA 5. If integers p and m are greater than zero, $0 \le \beta < p$ and $|\alpha| < \frac{\pi}{2}$, then

$$\prod_{j=0}^{m-1} \frac{\lambda^2|2(p-\beta)\cos\alpha e^{-i\alpha}+j|^2}{(j+1)^2} = \frac{\cos^2\alpha}{m^2}\{4\lambda^2(p-\beta)^2$$

$$+\sum_{k=1}^{m-1}[\lambda^2(2p-2\beta+k)^2+\lambda^2k^2\tan^2\alpha-k^2\sec^2\alpha]\times$$

$$\prod_{j=0}^{k-1}\frac{\lambda^2|2(p-\beta)\cos\alpha\, e^{-i\alpha}+j|^2}{(j+1)^2}\}. \tag{3.1}$$

PROOF. We prove the lemma by induction on m. For $m = 1$, (3.1) is easily verified directly.

Next suppose that (3.1) is true for $m = q-1$. We have

$$\frac{\cos^2\alpha}{q^2}\{4\lambda^2(p-\beta)^2+\sum_{k=1}^{q-1}[\lambda^2(2p-2\beta+k)^2+\lambda^2k^2\tan^2\alpha$$

$$-k^2\sec^2\alpha].\prod_{j=0}^{k-1}\frac{\lambda^2|2(p-\beta)\cos\alpha\, e^{-i\alpha}+j|^2}{(j+1)^2}\}$$

$$=\frac{\cos^2\alpha}{q^2}\{4\lambda^2(p-\beta)^2+\sum_{k=1}^{q-2}[\lambda^2(2p-2\beta+k)^2$$

$$+\lambda^2k^2\tan^2\alpha-k^2\sec^2\alpha]\prod_{j=0}^{k-1}\frac{\lambda^2|2(p-\beta)\cos\alpha\, e^{-i\alpha}+j|^2}{(j+1)^2}$$

$$+[\lambda^2(2p-2\beta+q-1)^2+\lambda^2(q-1)^2\tan^2\alpha-$$

$$(q-1)^2\sec^2\alpha]\prod_{j=0}^{q-2}\frac{\lambda^2|2(p-\beta)\cos\alpha\, e^{-i\alpha}+j|^2}{(j+1)^2}\}$$

$$=\prod_{j=0}^{q-2}\frac{\lambda^2|2(p-\beta)\cos\alpha\, e^{-i\alpha}+j|^2}{(j+1)^2}\times$$

$$\{\frac{\lambda^2(2p-2\beta+q-1)^2\cos^2\alpha+\lambda^2(q-1)^2\sin^2\alpha}{q^2}\}$$

$$=\prod_{j=0}^{q-1}\frac{\lambda^2|2(p-\beta)\cos\alpha\, e^{-i\alpha}+j|^2}{(j+1)^2}$$

Thus (3.1) holds for $m=q$ which proves lemma 5.

THEOREM 2. If $f(z) = z^p + \sum_{n=p+1}^{\infty} a_n z^n \in F_\lambda(\alpha,\beta,p)$, then

$$|a_n| \le \prod_{k=0}^{n-(p+1)} \frac{\lambda|2(p-\beta)\cos\alpha\, e^{-i\alpha}+k|}{k+1} \tag{3.2}$$

for $n \ge p+1$ and these bounds are sharp for all admissible α,β and λ for each n.

PROOF. As $f \in F_\lambda(\alpha,\beta,p)$, from Lemma 2, we have

$$\{e^{i\alpha}\sec\alpha\, zf'(z)+(p-2\beta-ip\tan\alpha)f(z)\}\, w(z)$$

$$=(p+ip\tan\alpha)f(z)-e^{-i\alpha}\sec\alpha zf'(z)$$

for $z \in U$, $w \in \Omega_\lambda$. Hence we have

$$\sum_{k=0}^{\infty}[\{(p+k)\, e^{i\alpha}\sec\alpha+(p-2\beta-ip\tan\alpha)\}\, a_{p+k}z^k]\, w(z)$$

$$=\sum_{k=0}^{\infty}[p+ip\tan\alpha-(p+k)e^{i\alpha}\sec\alpha]a_{p+k}z^k \tag{3.3}$$

where $a_p = 1$ and $w(z) = \sum_{k=0}^{\infty} b_{k+1}z^{k+1}$.

Equating coefficients of z^m on both sides of (3.3), we obtain

$$\sum_{k=0}^{m-1} \{(p+k)e^{i\alpha} \sec\alpha + (p-2\beta-ip \tan \alpha)\} a_{p+k} b_{m-k}$$

$$= \{p+ip \tan \alpha - (p+m)e^{i\alpha} \sec\alpha\}a_{p+m};$$

which shows that a_{p+m} on right hand side depends only on

$$a_p, a_{p+1}, \ldots, a_{p+(m-1)}$$

of left-hand side. Hence we can write

$$\sum_{k=0}^{m-1} [\{(p+k)e^{i\alpha} \sec\alpha + (p-2\beta-ip \tan \alpha)\} a_{p+k}z^k] w(z)$$

$$= \sum_{k=0}^{m} [p + ip \tan \alpha-(p+k)e^{i\alpha}\sec \alpha] a_{p+k} z^k + \sum_{k=m+1}^{\infty} A_k z^k \tag{3.4}$$

for $m = 1,2,3\ldots$ and a proper choice of A_k $(k \geq 0)$.

Denoting the right member of (3.4) by $G(z)$ and the factor multiplying $w(z)$ in the left member of (3.4) by $F(z)$, (3.4) assmes the form

$$G(z) = F(z) w(z) \quad \text{for} \quad z \in U.$$

Since $|w(z)| < \lambda$ for $z \in U$ this yields for $0 < r < 1$,

$$\frac{1}{2\pi} \int_0^{2\pi} |G(re^{i\theta})|^2 d\theta \leq \frac{\lambda^2}{2\pi} \cdot \int_0^{2\pi} |F(re^{i\theta})|^2 d\theta,$$

hence, using the definitions of $G(z)$ and $F(z)$

$$\sum_{k=0}^{m} |p+ip \tan \alpha - (p+k) e^{i\alpha} \sec\alpha|^2 |a_{p+k}|^2 r^{2k}$$

$$+ \sum_{k=m+1}^{\infty} |A_k|^2 r^{2k} \leq$$

$$\lambda^2\{\sum_{k=0}^{m-1} |(p+k)e^{i\alpha} \sec\alpha + (p-2\beta-ip \tan \alpha)|^2 |a_{p+k}|^2 r^{2k}\}. \tag{3.5}$$

Setting $r \to 1$ in (3.5), the inequality (3.5) may be written as

$$\sum_{k=0}^{m-1} \{\lambda^2 |(p+k)e^{i\alpha} \sec\alpha + (p-2\beta-ip \tan \alpha)|^2 -$$

$$|p +ip \tan \alpha - (p+k)e^{i\alpha} \sec \alpha|^2\} |a_{p+k}|^2$$

$$\geq |p+ip \tan \alpha - (p+m)e^{i\alpha} \sec \alpha|^2 |a_{p+m}|^2 . \tag{3.6}$$

Simplification of (3.6) leads to

$$|a_{p+m}|^2 \leq \frac{\cos^2\alpha}{m^2} \cdot \sum_{k=0}^{m-1} \{\lambda^2(2p-2\beta+k)^2 +$$

$$\lambda^2 k^2 \tan^2\alpha - k^2 \sec^2\alpha\} |a_{p+k}|^2. \tag{3.7}$$

Replacing $p+m$ by n in (3.7), we are led to

$$|a_n|^2 \leq \frac{\cos^2\alpha}{(n-p)^2} \cdot \sum_{k=0}^{n-(p+1)} \{\lambda^2(2p-2\beta+k)^2 +$$

$$\lambda^2k^2 \tan^2\alpha - k^2 \sec^2\alpha\} |a_{p+k}|^2 \tag{3.8}$$

where $n \geq p + 1$.

For $n = p + 1$, (3.8) reduces to

$$|a_{p+1}|^2 \leq 4(p-\beta)^2 \lambda^2 \cos^2 \alpha$$

or

$$|a_{p+1}| \leq 2(p-\beta) \lambda \cos \alpha \tag{3.9}$$

which is equivalent to (3.2).

To establish (3.2) for $n > p+1$, we will apply induction argument.

Fix n, $n \geq p + 2$, and suppose (3.2) holds for $k = 1,2,\ldots, n-(p+1)$. Then

$$|a_n|^2 \leq \frac{\cos^2\alpha}{(n-p)^2}\{4\lambda^2(p-\beta)^2 + \sum_{k=0}^{n-(p+1)} [\lambda^2(2p-2\beta+k)^2 + \lambda^2k^2 \tan^2 \alpha - k^2 \sec^2\alpha] \times \prod_{j=0}^{k-1} \frac{\lambda^2|2(p-\beta)\cos \alpha\, e^{-i\alpha}+j|^2}{(j+1)^2} \} \tag{3.10}$$

Thus from (3.8), (3.10) and Lemma 5 with $m = n - p$, we obtain

$$|a_n|^2 \leq \prod_{j=0}^{n-(p+1)} \frac{\lambda^2|2(p-\beta)\cos\alpha\, e^{-i\alpha}+j|^2}{(j+1)^2}.$$

This completes the proof of Theorem 2.

Equality holds in (3.2) for $n \geq P + 1$ for the function $f(z) \in A_p$ defined by (2.1) with $w(z) = \lambda z$.

REMARK ON THEOREM 2. For various choices of the parameters, known results can be regained: [7], [8], [12], [13], [14], [16] [20].

In a similar way we can prove the following: Lemma 6, 7, and Theorem 3 for functions in $C_\lambda(b,p)$.

4. REPRESENTATION FORMULAS FOR THE CLASS $C_\lambda(b,p)$

LEMMA 6. $g(z) \in C_\lambda(b,p)$ if and only if for $z \in U$

(i) $$\frac{zg''(z)}{g'(z)} = \frac{(p-1)+(p-2pb-1)w(z)}{1+w(z)}, \quad w \in \Omega_\lambda. \tag{4.1}$$

(ii) $$g'(z) = pz^{p-1} [\frac{g_1(z)}{z}]^{pb} \tag{4.2}$$

for some $g_1 \in S(\lambda)$.

(iii) $$g'(z) = pz^{p-1} \exp[-2pb \int_0^z \frac{\phi(t)}{1+t\,\phi(t)} dt], \tag{4.3}$$

where $\phi(z)$ is analytic and satisfies $|\phi(z)| \leq \lambda$, $0 < \lambda < 1$, for $|z| < 1$.

5. COEFFICIENT ESTIMATES FOR THE CLASS $C_\lambda(b,p)$.

LEMMA 7. If integers p and m are greater than zero; $b \neq 0$ and complex, then

$$\prod_{j=0}^{m-1} \frac{\lambda^2|2pb+j|^2}{(j+1)^2} = \frac{1}{m^2}\{4 p^2|b|^2 \cdot \lambda^2 + \sum_{k=1}^{m-1} (k^2(\lambda^2-1) + 4p^2|b|^2 \lambda^2 + 4pk \operatorname{Re}\{b\}\lambda^2) \prod_{j=0}^{k-1} \frac{\lambda^2|2pb+j|^2}{(j+1)^2}\}. \tag{5.1}$$

THEOREM 3. If $g(z) = z^p + \sum_{n=p+1}^{\infty} d_n z^n \in C_\lambda(b,p)$, then

$$|d_n| \leq \frac{p}{n} \cdot \prod_{k=0}^{n-(p+1)} \frac{\lambda|2pb+k|}{(k+1)} \tag{5.2}$$

for $n \geq p+1$. Equality holds in (5.2) for the function $g(z) \in A_p$ defined by (4.1) with $w(z) = \lambda z$.

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As a multidisciplinary field, financial engineering is becoming increasingly important in today's economic and financial world, especially in areas such as portfolio management, asset valuation and prediction, fraud detection, and credit risk management. For example, in a credit risk context, the recently approved Basel II guidelines advise financial institutions to build comprehensible credit risk models in order to optimize their capital allocation policy. Computational methods are being intensively studied and applied to improve the quality of the financial decisions that need to be made. Until now, computational methods and models are central to the analysis of economic and financial decisions.

However, more and more researchers have found that the financial environment is not ruled by mathematical distributions or statistical models. In such situations, some attempts have also been made to develop financial engineering models using intelligent computing approaches. For example, an artificial neural network (ANN) is a nonparametric estimation technique which does not make any distributional assumptions regarding the underlying asset. Instead, ANN approach develops a model using sets of unknown parameters and lets the optimization routine seek the best fitting parameters to obtain the desired results. The main aim of this special issue is not to merely illustrate the superior performance of a new intelligent computational method, but also to demonstrate how it can be used effectively in a financial engineering environment to improve and facilitate financial decision making. In this sense, the submissions should especially address how the results of estimated computational models (e.g., ANN, support vector machines, evolutionary algorithm, and fuzzy models) can be used to develop intelligent, easy-to-use, and/or comprehensible computational systems (e.g., decision support systems, agent-based system, and web-based systems)

This special issue will include (but not be limited to) the following topics:

- **Computational methods**: artificial intelligence, neural networks, evolutionary algorithms, fuzzy inference, hybrid learning, ensemble learning, cooperative learning, multiagent learning
- **Application fields**: asset valuation and prediction, asset allocation and portfolio selection, bankruptcy prediction, fraud detection, credit risk management
- **Implementation aspects**: decision support systems, expert systems, information systems, intelligent agents, web service, monitoring, deployment, implementation

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