

M-QUASI-HYPONORMAL COMPOSITION OPERATORS

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ABSTRACT. A necessary and sufficient condition is obtained for M-quasi-hyponormal composition operators. It has also been proved that the class of M-quasi-hyponormal composition operators coincides with the class of M-paranormal composition operators. Existence of M-hyponormal composition operators which are not hyponormal; and M-quasi-hyponormal composition operators which are not M-hyponormal and quasi-hyponormal are also shown.

KEY WORDS AND PHRASES. M-hyponormal, M-quasi-hyponormal, M-paranormal, normal composition operators.

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1. PRELIMINARIES.

Let (X, S, m) be a sigma-finite measure space and T a measurable transformation from X into itself (that is one $mT^{-1}(E) = 0$ whenever $m(E) = 0$ for $E \in S$). Then the equation $C_T f = f \circ T$ for every f in $L^2(m)$ defines a linear transformation. If C_T is bounded with range in $L^2(m)$, then it is called composition operator. If $X = N$ the set of all non-zero positive integers and m is counting measure on the family of all subsets of N , then $L^2(m) = \ell^2$ (the Hilbert space of all square summable sequences).

Let $f_0 = \frac{dmT^{-1}}{dm}$ be the Radon-Nikodym derivative of the measure mT^{-1} with respect to the measure m ,

$$\frac{dm(ToT)^{-1}}{dmT^{-1}} = g_0, \quad \frac{dm(ToT)^{-1}}{dm} = h_0$$

Then $h_0 = f_0 \cdot g_0$.

Let $B(H)$ denote the Banach algebra of all bounded linear operators on the Hilbert space H . An operator $T \in B(H)$ is called M-quasi-hyponormal if there exists $M > 0$ such that

$$M^2 T^{*2} T^2 - (T^* T)^2 \geq 0$$

or equivalently $||T^*Tx|| \leq M ||T^2x||$ for all x in H [1]. T is said to be M -paranormal [2] if for all unit vectors x in H

$$||Tx||^2 \leq M ||T^2x||.$$

T is said to be M -hyponormal [2] if

$$||Tx|| \leq M ||T^*Tx|| \quad \text{for all } x \text{ in } H.$$

The purpose of this paper is to generalize the results on quasi-hyponormal composition operators in [3] for M -quasi-hyponormal composition operators.

2. M -QUASI-HYPONORMAL COMPOSITION OPERATORS.

In this section we obtain a necessary and sufficient condition for M -quasi-hyponormal composition operators and then show that the class of M -quasi-hyponormal composition operators on ℓ^2 coincides with the class of M -paranormal composition operators. We also show the existence of M -hyponormal composition operators which are not hyponormal, and M -quasi-hyponormal composition operators which are not M -hyponormal and quasi-hyponormal.

THEOREM 2.1. Let $C_T \in B(\ell^2)$. Then C_T is M -quasi-hyponormal if and only if $f_o^2 \leq M^2 h_o$.

PROOF. Since for any f in L^2 ,

$$\begin{aligned} (C_T^* C_T^2 f, f) &= (C_T^2 f, C_T^2 f) = \int h_o |f|^2 dm, \\ &= (M_{h_o} f, f), \end{aligned}$$

where M_{h_o} is the multiplication operator induced by h_o , therefore $C_T^* C_T^2 = M_{h_o}$.

Similarly it can be seen that $C_T^* C_T = M_{f_o}$. C_T is M -quasi-hyponormal if and only if

$$M^2 C_T^* C_T^2 - (C_T^* C_T)^2 \geq 0.$$

This implies that

$$M^2 M_{h_o} - M_{f_o}^2 \geq 0,$$

that is $f_o^2 \leq M^2 h_o$.

Hence the result.

COROLLARY. Let $C_T \in B(\ell^2)$. Then C_T is M -quasi-hyponormal if and only if $f_o \leq M^2 g_o$.

PROOF. Since $h_o = f_o \cdot g_o$ and f_o is positive, therefore, by above theorem we get the result.

THEOREM 2.2. Let $C_T \in B(\ell^2)$. Then C_T is M -quasi-hyponormal if and only if C_T is M -paranormal.

PROOF. Necessity is true for any bounded operator A . For the sufficiency, let C_T be M -paranormal, then

$$||C_T X_{\{n\}}||^2 \leq M ||C_T^2 X_{\{n\}}||^2 \quad \text{for all } n \in N$$

$$\text{or } \int |X_{\{n\}} \circ T|^2 dm \leq M (\int |X_{\{n\}} \circ T|^2 dm)^{1/2}$$

$$\text{or } \int |X_{\{n\}}|^2 dm T^{-1} \leq M (\int |X_{\{n\}}|^2 dm (ToT)^{-1})^{1/2}$$

$$\text{or } \int_{\{n\}} f_o \, dm \leq M (\int_{\{n\}} h_o \, dm)^{1/2}$$

$$\text{or } f_o^2(n) \leq M^2 h_o(n) \quad \text{for all } n \text{ in } N.$$

Hence $f_o^2 \leq M^2 h_o$; C_T is M-quasi-hyponormal.

THEOREM 2.3. Let $C_T \in B(\ell^2)$ and $T:N \rightarrow N$ be one-to-one. Then the following are equivalent.

- (i) Normal
- (ii) M-hyponormal
- (iii) M-quasi-hyponormal.

PROOF. (i) implies (ii), (ii) implies (iii) are always true for any bounded operator A. We show that (iii) implies (i). Let C_T be M-quasi-hyponormal. Then $||C_T^* C_T f|| \leq M ||C_T^2 f||$ for all f in ℓ^2 . Now T is onto because if T is not onto then $N \setminus T(N)$ is non-empty and for $n \in N \setminus T(N)$

$$||C_T^* C_T X_{\{n\}}|| = 1 \quad \text{and} \quad ||C_T C_T X_{\{n\}}|| = 0.$$

There exists no $M > 0$ such that C_T is M-quasi-hyponormal which is a contradiction.

Since T is one-to-one, therefore, T is invertible, by Theorem 2.2 [4] C_T is invertible and C_T is normal by Theorem 2.1 [3].

Here we give an example of a composition operator on ℓ^2 which is M-hyponormal but not hyponormal.

EXAMPLE 1. Let $T:N \rightarrow N$ be the mapping such that

$$T(1) = 2, \quad T(2) = 1, \quad T(3) = 2 \quad \text{and}$$

$$T(3n+m) = n+2, \quad m = 1, 2, 3 \quad \text{and } n \in N.$$

Then C_T is not hyponormal as $f_o \circ T \not\leq f_o$ for $n = 1$. C_T is M-hyponormal for $M \geq \sqrt{2}$.

EXAMPLE 2. Let $T:N \rightarrow N$ be defined by $T(1) = 2, T(2) = 1, T(3n+m) = n+1, m = 0, 1, 2$ and $n \in N$. Then C_T is $\sqrt{2}$ -quasi-hyponormal but C_T is not $\sqrt{2}$ -hyponormal. C_T is not quasi-hyponormal also.

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