

Research Article

An Extension of Gregus Fixed Point Theorem

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Let C be a closed convex subset of a complete metrizable topological vector space (X, d) and $T : C \rightarrow C$ a mapping that satisfies $d(Tx, Ty) \leq ad(x, y) + bd(x, Tx) + cd(y, Ty) + ed(y, Tx) + fd(x, Ty)$ for all $x, y \in C$, where $0 < a < 1$, $b \geq 0$, $c \geq 0$, $e \geq 0$, $f \geq 0$, and $a + b + c + e + f = 1$. Then T has a unique fixed point. The above theorem, which is a generalization and an extension of the results of several authors, is proved in this paper. In addition, we use the Mann iteration to approximate the fixed point of T .

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1. Introduction

Gregus [1] proved the following theorem.

THEOREM 1.1. *Let C be a closed convex subset of a Banach space X and $T : C \rightarrow C$ a mapping that satisfies $\|Tx - Ty\| \leq a\|x - y\| + b\|x - Tx\| + c\|y - Ty\|$ for all $x, y \in C$, where $0 < a < 1$, $b \geq 0$, $c \geq 0$, and $a + b + c = 1$. Then T has a unique fixed point.*

Several papers have been written on the Gregus fixed point theorem. For example, see [2, 3]. The theorem has been generalized to the condition when X is a complete metrizable topological vector space [4].

When $a = 1$, $b = 0$, $c = 0$, T becomes a nonexpansive map. In the past four decades, several papers have been written on the existence of a fixed point (which may not be unique) for a nonexpansive map defined on a closed bounded and convex subset C of a Banach space X . For example, see [5–7]. Recently, the existence of fixed points of T when the domain of T is unbounded was discussed in [6]. When $a = 0$, we have the Kannan maps. Similarly, several papers have been written on the existence of a fixed point for a

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Kannan map defined on a Banach space, for example, see [8, 9]. The fixed point theorem of Gregus is interesting because it tells what happens if $0 < a < 1$.

Chatterjea [10] considered the existence of fixed point for T when T is defined on a metric space (X, d) , such that for $0 < a < 1/2$,

$$d(Tx, Ty) \leq a\{d(x, f(y)) + d(y, f(x))\}. \quad (1.1)$$

It is natural to combine this condition with that of Gregus to get the following condition:

$$d(Tx, Ty) \leq ad(x, y) + bd(x, Tx) + cd(y, Ty) + ed(y, Tx) + fd(x, Ty) \quad (1.2)$$

for all $x, y \in C$, where $0 < a < 1$, $b \geq 0$, $c \geq 0$, $e \geq 0$, $f \geq 0$, and $a + b + c + e + f = 1$.

Observe that if T satisfies (1.2), then it also satisfies

$$d(Tx, Ty) \leq ad(x, y) + pd(x, Tx) + pd(y, Ty) + pd(y, Tx) + pd(x, Ty) \quad (1.3)$$

for all $x, y \in C$, where $0 < a < 1$, $p \geq 0$, $a + 4p = 1$, ($p = (1/4)b + (1/4)c + (1/4)e + (1/4)f$). Thus b , c , e , and f will be used interchangeably as p in the proof of our main theorem.

As observed by Chidume [5, page 119], since the four points $\{x, y, Tx, Ty\}$ of (1.2) determine six distances in X , the inequality amounts to say that the image distance $d(Tx, Ty)$ never exceeds a fixed convex combination of the remaining five distances. Geometrically, this type of condition is quite natural.

In this paper, we extend Gregus result to the condition when T satisfies condition (1.2) and also generalize it to the condition when X is a complete metrizable topological vector space, thus answering the question posed in [4]. Complete metrizable topological vector spaces include uniformly convex Banach spaces, Banach spaces and complete metrizable locally convex spaces (see [11, 12]).

The following result will be needed for our result.

THEOREM 1.2 [13, 14]. *A topological vector space X is metrizable if and only if it has a countable base of neighbourhoods of zero. The topology of a metrizable topological vector space can always be defined by a real-valued function $\|\cdot\| : X \rightarrow \mathbb{R}$, called F -norm such that for all $x, y \in X$,*

- (1) $\|x\| \geq 0$,
- (2) $\|x\| = 0 \Rightarrow x = 0$,
- (3) $\|x + y\| \leq \|x\| + \|y\|$,
- (4) $\|\lambda x\| \leq \|x\|$ for all $\lambda \in K$ with $|\lambda| \leq 1$,
- (5) if $\lambda_n \rightarrow 0$, and $\lambda_n \in K$, then $\|\lambda_n x\| \rightarrow 0$.

For the same result see Kothe [15, Section 15.11]. Henceforth, unless otherwise indicated, F will denote an F -norm if it is characterizing a metrizable topological vector space. Observe that an F -norm will be a norm if it is defining a normed space.

We now prove our main theorem. We use the technique in [4] which is due to Gregus [1].

THEOREM 1.3. *Let C be a closed convex subset of a complete metrizable space X and $T : C \rightarrow C$ a mapping that satisfies $F(Tx - Ty) \leq aF(x - y) + bF(x - Tx) + cF(y - Ty) + eF(y - Tx) + fF(x - Ty)$ for all $x, y \in C$, where $0 < a < 1$, $b \geq 0$, $c \geq 0$, $e \geq 0$, $f \geq 0$, and $a + b + c + e + f = 1$. Then T has a unique fixed point.*

Proof. Take any point $x \in C$ and consider the sequence $\{T_n(x)\}_{n=1}^\infty$,

$$\begin{aligned}
 F(T^n x - T^{n-1} x) &\leq aF(T^{n-1} x - T^{n-2} x) + bF(T^{n-1} x - T^n x) \\
 &\quad + cF(T^{n-2} x - T^{n-1} x) + eF(T^{n-2} x - T^n x) \\
 &\quad + fF(T^{n-1} x - T^{n-1} x) \\
 &\leq \frac{a+c+e}{1-b-e} F(T^{n-1} x - T^{n-2} x) \\
 &\leq \frac{a+2p}{1-2p} F(T^{n-1} x - T^{n-2} x) \leq F(Tx - x).
 \end{aligned} \tag{1.4}$$

Thus

$$F(T^n x - T^{n-1} x) \leq F(Tx - x). \tag{1.5}$$

In effect, it means that the distance between two consecutive elements of $\{T^n x\}$ is less or equal to the distance between the first and the second element. Now let us consider the distance between two consecutive elements with odd (resp., even) power of T . It is sufficient to consider only the distance between Tx and $T^3 x$,

$$\begin{aligned}
 F(T^3 x - Tx) &\leq aF(T^2 x - x) + bF(T^2 x - T^3 x) + cF(Tx - x) \\
 &\quad + eF(x - T^3 x) + fF(T^2 x - Tx) \\
 &\leq aF(T^2 x - Tx) + aF(Tx - x) + bF(T^2 x - T^3 x) \\
 &\quad + cF(Tx - x) + eF(x - Tx) + eF(Tx - T^2 x) \\
 &\quad + eF(T^2 x - T^3 x) + fF(T^2 x - Tx) \\
 &\leq (2a + b + c + 3e + f)F(Tx - x) = (a + 2p + 1)F(Tx - x).
 \end{aligned} \tag{1.6}$$

Hence

$$F(T^3 x - Tx) \leq (a + 2p + 1)F(Tx - x) \quad \forall x \in C. \tag{1.7}$$

Since C is convex, therefore $z = (1/2)T^2 x + (1/2)T^3 x$ is in C , and from the properties of the F -norm, we have

$$\begin{aligned}
 F(Tz - z) &\leq \frac{1}{2}F(Tz - T^2 x) + \frac{1}{2}F(Tz - T^3 x) \\
 &\leq \frac{1}{2}\{aF(z - Tx) + bF(Tz - z) + cF(Tx - T^2 x) \\
 &\quad + eF(Tx - Tz) + fF(z - T^2 x)\} \\
 &\quad + \frac{1}{2}\{aF(z - T^2 x) + bF(Tz - z) + cF(T^3 x - T^2 x) \\
 &\quad + eF(T^2 x - Tz) + fF(z - T^3 x)\},
 \end{aligned}$$

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$$\begin{aligned}
 F(z - Tx) &\leq \frac{1}{2}F(T^2x - Tx) + \frac{1}{2}F(T^3x - Tx) \\
 &\leq \frac{1}{2}F(Tx - x) + \frac{1}{2}(a + 2p + 1)F(Tx - x) = \left(1 + p + \frac{1}{2}a\right)F(Tx - x), \\
 F(z - T^2x) &\leq \frac{1}{2}F(T^3x - T^2x) \leq \frac{1}{2}F(Tx - x).
 \end{aligned} \tag{1.8}$$

Similarly,

$$\begin{aligned}
 F(z - T^3x) &\leq \frac{1}{2}F(Tx - x), \\
 F(Tx - Tz) &\leq \frac{1}{2}F(Tx - T^3x) + \frac{1}{2}F(Tx - T^4x) \\
 &\leq \frac{1}{2}(a + 2p + 1)F(Tx - x) + \frac{1}{2}\{F(Tx - T^2x) + F(T^2x - T^4x)\} \\
 &\leq \frac{1}{2}(a + 2p + 1)F(Tx - x) + \frac{1}{2}\{F(Tx - x) + (a + 2p + 1)F(Tx - x)\} \\
 &\leq \left(a + 2p + \frac{3}{2}\right)F(Tx - x), \\
 F(T^2x - Tz) &\leq \frac{1}{2}F(T^2x - T^3x) + \frac{1}{2}F(T^2x - T^4x) \leq \left(\frac{1}{2}a + p + 1\right)F(Tx - x).
 \end{aligned} \tag{1.9}$$

Thus

$$\begin{aligned}
 (1 - b)F(Tz - z) &\leq \frac{1}{2}\left\{a\left(1 + p + \frac{1}{2}a\right)F(Tx - x) + cF(Tx - x)\right. \\
 &\quad \left.+ e\left(a + 2p + \frac{3}{2}\right)F(Tx - x) + \frac{1}{2}fF(Tx - x)\right\} \\
 &\quad + \frac{1}{2}\left\{\frac{1}{2}aF(Tx - x) + cF(Tx - x) + \frac{1}{2}e(a + 2p + 1)F(Tx - x)\right. \\
 &\quad \left.+ \frac{1}{2}fF(Tx - x)\right\} = \left(\frac{3}{4}a + \frac{1}{4}a^2 + \frac{5}{4}ap + \frac{5}{2}p + \frac{3}{2}p^2\right)F(Tx - x).
 \end{aligned} \tag{1.10}$$

Thus

$$\begin{aligned}
 4(1 - p)F(z - Tz) &\leq (3a + a^2 + 5ap + 10p + 6p^2)F(Tx - x) \\
 &\leq (2p^2 - 5p + 4)F(Tx - x).
 \end{aligned} \tag{1.11}$$

Hence

$$\begin{aligned}
 F(z - Tz) &\leq \frac{26 - 22a - a^2}{8(a + 3)}F(Tx - x), \\
 F(Tz - z) &\leq \lambda F(Tx - x),
 \end{aligned} \tag{1.12}$$

where $\lambda = (26 - 22a - a^2)/8(a + 3)$. It is clear that $0 < \lambda < 1$.

Now let $i = \inf\{F(Tx - x) : x \in C\}$. Then there exists a point $x \in C$ such that $F(Tx - x) < i + \epsilon$ for $\epsilon > 0$.

Suppose $i > 0$. Then for $0 < \epsilon < (1 - \lambda)i/\lambda$ and $F(Tx - x) < i + \epsilon$, we have

$$F(Tz - z) \leq \lambda F(Tx - x) \leq \lambda(i + \epsilon) < i, \quad (1.13)$$

that is, $F(Tz - z) < i$, which is a contradiction with the definition of i . Hence $\inf\{F(Tx - x) : x \in C\} = 0$.

To prove that the infimum is attained is the easy part of the proof. Take the following system of sets: $K_n = \{x : F(x - Tx) \leq 1/2n(q + 1)\}$; $T(K_n)$ and $\overline{T(K_n)}$, where $n \in \mathbb{N}$, $q = (a + p)/(1 - a)$, and $\overline{T(K_n)}$ is the closure of $T(K_n)$. Then for any $x, y \in K_n$,

$$\begin{aligned} F(Tx - Ty) &\leq qF(Tx - x) + qF(Ty - y) \leq \frac{1}{n}, \\ F(x - y) &\leq (q + 1)F(Tx - x) + (q + 1)F(Ty - y) \leq \frac{1}{n}, \end{aligned} \quad (1.14)$$

that is, $\text{diam}(K_n) \leq 1/n$, $\text{diam}(T(K_n)) \leq 1/n$ and therefore, since $\text{diam}(T(K_n)) = \text{diam}(\overline{T(K_n)})$, we have $\text{diam}(\overline{T(K_n)}) \leq 1/n$. It is clear that $\{K_n\}$ and $\{\overline{T(K_n)}\}$ form monotone sequences of sets and from (1.5) we have $T(K_n) \subset K_n$. Suppose $y \in \overline{T(K_n)}$, then there exists $y' \in K_n$ such that $F(y - Ty') < \epsilon$ for $\epsilon > 0$ and

$$\begin{aligned} F(y - Ty) &\leq F(y - Ty') + F(Ty' - Ty) \\ &\leq F(y - Ty') + aF(y - y') + bF(y' - Ty') \\ &\quad + cF(Ty - y) + eF(y - Ty') + fF(y' - Ty). \end{aligned} \quad (1.15)$$

Hence

$$(1 - c)F(y - Ty) \leq (1 + a + e + f)\epsilon + (a + b)F(Ty' - y'). \quad (1.16)$$

Since $F(y' - Ty') \leq 1/2n(q + 1)$, then

$$F(y - Ty) \leq \frac{1 + a + e + f}{1 - c}\epsilon + \frac{a + b}{1 - c} \frac{1}{2n(q + 1)}. \quad (1.17)$$

Since $\epsilon > 0$ is arbitrary and $a + b + c \leq 1$, then $F(y - Ty) \leq 1/2n(q + 1)$ and we have $y \in K_n$. Hence $\overline{T(K_n)} \subset K_n$, too.

$\{\overline{T(K_n)}\}$ is a decreasing sequence of closed nonempty sets with $\text{diam}(\overline{T(K_n)}) \rightarrow 0$ as $n \rightarrow \infty$. Hence they have a nonempty intersection $\{x^*\}$ and T has a unique fixed point $Tx^* = x^*$. \square

COROLLARY 1.4. *Let C be a closed convex subset of a Banach space X and $T : C \rightarrow C$ a mapping that satisfies $\|Tx - Ty\| \leq a\|x - y\| + b\|Tx - x\| + c\|Ty - y\| + e\|Tx - y\| + f\|Ty - x\|$ for all $x, y \in C$ where $0 < a < 1$, $b \geq 0$, $c \geq 0$, $e \geq 0$, $f \geq 0$, and $a + b + c + e + f = 1$. Then T has a unique fixed point.*

COROLLARY 1.5 [1]. *Let C be a closed convex subset of a Banach space X and $T : C \rightarrow C$ a mapping that satisfies $\|Tx - Ty\| \leq a\|x - y\| + b\|Tx - x\| + c\|Ty - y\|$ for all $x, y \in C$, where $0 < a < 1$, $b \geq 0$, $c \geq 0$, and $a + b + c = 1$. Then T has a unique fixed point.*

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COROLLARY 1.6. *Let C be a closed convex subset of a complete metrizable topological vector space X and $T : C \rightarrow C$ a mapping that satisfies $\|Tx - Ty\| \leq a\|x - y\| + b\|Tx - y\| + c\|Ty - x\|$ for all $x, y \in C$, where $0 < a < 1$, $b \geq 0$, $c \geq 0$, and $a + b + c = 1$. Then T has a unique fixed point.*

We now proceed to use the Mann iteration scheme [16] to approximate the fixed point of our mapping under consideration.

THEOREM 1.7. *Let C be a nonempty closed convex subset of a complete metrizable topological vector space X and let $T : C \rightarrow C$ be a mapping that satisfies $F(Tx - Ty) \leq aF(x - y) + bF(Tx - x) + cF(Ty - y) + eF(Tx - y) + fF(Ty - x)$ for all $x, y \in C$, where $0 < a < 1$, $b \geq 0$, $c \geq 0$, $e \geq 0$, $f \geq 0$, and $a + b + c + e + f = 1$. Suppose $\{x_n\}$ is a Mann iteration sequence defined by $x_{n+1} = (1 - \alpha_n)x_n + \alpha_nTx_n$, $x_0 \in C$, $n \geq 0$, where $\{\alpha_n\}$ satisfy $0 < \alpha_n \leq 1$ for all n , $\sum_0^\infty \alpha_n = \infty$. Assume $2c < c + b$, then $\{x_n\}$ converges to the unique fixed point of T .*

Proof. The fact that T has a unique fixed point is already shown in Theorem 1.3.

If $F(Tx - Ty) \leq aF(x - y) + bF(Tx - x) + cF(Ty - y) + eF(Tx - y) + fF(Ty - x)$, then

$$\begin{aligned} F(Tx - Ty) &\leq aF(x - y) + bF(Tx - x) + c\{F(Ty - Tx) + F(Tx - x) + F(x - y)\} \\ &\quad + e\{F(Tx - x) + F(x - y)\} + f\{F(Ty - Tx) + F(Tx - x)\}. \end{aligned} \quad (1.18)$$

After computation, we have $F(Tx - Ty) \leq ((a + c + e)/(1 - (c + f)))F(x - y) + ((b + c + e + f)/(1 - (c + f)))F(Tx - x)$. If $\delta = (a + c + e)/(1 - (c + f))$, then

$$F(Tx - Ty) \leq \delta F(x - y) + \frac{b + c + e + f}{1 - (c + f)} F(Tx - x). \quad (1.19)$$

Since by assumption $2c < b + c$, it is clear that $\delta < 1$.

Suppose p is a fixed point of T , then if $x = p$ and $y = x_n$, from (1.19), we obtain

$$\begin{aligned} F(Tx_n - p) &\leq \delta F(x_n - p), \\ F(x_{n+1} - p) &= F((1 - \alpha_n)x_n + \alpha_nTx_n - (1 - \alpha_n + \alpha_n)p) \\ &= F((1 - \alpha_n)(x_n - p) + \alpha_n(Tx_n - p)) \\ &\leq (1 - \alpha_n)F(x_n - p) + \alpha_nF(Tx_n - p) \\ &\leq (1 - \alpha_n(1 - \delta))F(x_n - p). \end{aligned} \quad (1.20)$$

Since $1 - \alpha_n(1 - \delta) < 1$ by the choice of α_n in the theorem, then $\{x_n\}$ converges to p . \square

Remarks 1.8. (1) Gregus [1] gave an example in which $a = 1$, C is closed convex and bounded but yet T does not have a fixed point. If $a = 1$, some form of boundedness must be assumed on C for T to have a fixed point, for example, see [7, 6]. The same is true if $a = 0$ (see [8, 9]).

(2) If (X, d) is a complete metric space and $a + b + c + e + f < 1$, it was shown in [17] that T as defined in (1.2) has a unique fixed point. However, if $a + b + c + e + f = 1$, Hardy

and Rogers [17] assumed that T is continuous and X is compact in order to prove the existence of fixed point for T as defined in (1.2). Goebel et al. [18] obtained the existence of fixed point for T as defined by (1.2) when $a + b + c + e + f = 1$. In which case, it was assumed that X is a uniformly convex Banach space, T is continuous and C is bounded, closed, and convex. In our result, T is not assumed to be continuous, X is assumed to be neither a compact nor a uniformly convex Banach space, and there is no boundedness assumption on C .

(3) Berinde [14] showed that the Ishikawa iteration sequence [16] of a class of quasi-contractive operators, called Zamfirescu operators, defined on a closed convex subset C of a Banach space X converges to the fixed point of T . The first author [19] showed that if X is a complete metrizable locally convex space, and C is closed and convex, then the Mann iteration sequence of the Zamfirescu operator T defined on C converges to the fixed point of T . In both cases, the sum of the constants is less than 1 while in Theorem 1.7, the sum is 1. In addition, X is generalized to a complete metrizable topological vector spaces. Can Theorem 1.7 still be proved without the assumption that $2c < a + b$?

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