

EXISTENCE OF POSITIVE SOLUTION FOR SECOND-ORDER IMPULSIVE BOUNDARY VALUE PROBLEMS ON INFINITY INTERVALS

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We deal with the existence of positive solutions to impulsive second-order differential equations subject to some boundary conditions on the semi-infinity interval.

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1. Introduction

In recent years, impulsive differential equations have become a very active area of research and we refer the reader to the monographs [8] and the articles [6, 9, 10, 14, 15], where properties of their solutions are studied and extensive bibliographies are given. In consequence, it is very important to develop a complete basic theory of impulsive differential equations. Also, infinite interval problems have been extensively studied, see [1–5, 11, 12].

In this paper we study the existence of positive solutions for the following boundary value problem (BVP) with impulses:

$$\begin{aligned}y'' + g(t, y, y') &= 0, \quad 0 < t < \infty, t \neq t_k, \\ \Delta y'(t_k) &= b_k y'(t_k), \quad \Delta y(t_k) = a_k y(t_k), \quad k = 1, 2, \dots, \\ y(0) &= 0, \quad y \text{ bounded on } [0, \infty),\end{aligned}\tag{1.1}$$

where $t_k < t_{k+1}$, $\lim_{k \rightarrow \infty} t_k = \infty$, $\Delta y'(t_k) = y'(t_k^+) - y'(t_k^-)$, $\Delta y(t_k) = y(t_k^+) - y(t_k^-)$, and g is continuous except $\{t_k\} \times \mathbb{R} \times \mathbb{R}$; we assume that for $k \in \mathbb{N}^+ = \{1, 2, \dots\}$ and $x, y \in \mathbb{R}$ there exist the limits

$$\lim_{t \rightarrow t_k^-} g(t, x, y) = g(t_k, x, y), \quad \lim_{t \rightarrow t_k^+} g(t, x, y).\tag{1.2}$$

The problems of the above type without impulses have been discussed by several authors in the literature, we refer the reader to the pioneer works of Agarwal and O'Regan [1, 2, 4] and Ma [12] and Constantin [11]. But as far as we know the publication on solvability of infinity interval problems with impulses is fewer [15]. In this paper we want to

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fill in this gap and extend the existence results on the case of infinity interval problems with impulses.

Motivated by works of [2, 12], we use the well-known Leray-Schauder continuation theorem [13] to establish new results on finite intervals $[0, n]$ and use a diagonalization argument to get positive solutions on infinity intervals.

Let $J = [0, a]$, a is a constant or $a = +\infty$, in order to define the concept of solution for BVP (1.1), we introduce the following spaces of functions:

$PC(J) = \{u : J \rightarrow \mathbb{R}, u \text{ is continuous at } t \neq t_k, u(t_k^+), u(t_k^-) \text{ exist, and } u(t_k^-) = u(t_k)\};$

$PC^1(J) = \{u \in PC(J) : u \text{ is continuously differentiable at } t \neq t_k, u'(0^+), u'(t_k^+), u'(t_k^-) \text{ exist, and } u'(t_k^-) = u'(t_k)\};$

$PC^2(J) = \{u \in PC^1(J) : u \text{ is twice continuously differentiable at } t \neq t_k\}.$

Note that $PC(J)$ and $PC^1(J)$ are Banach spaces with the norms

$$\|u\|_\infty = \sup \{ |u(t)| : t \in J \}, \quad \|u\|_1 = \max \{ \|u\|_\infty, \|u'\|_\infty \}, \quad (1.3)$$

respectively.

Definition 1.1. By a positive solution of BVP (1.1), one means a function $y(t)$ satisfying the following conditions:

- (i) $y \in PC^1[0, \infty)$;
- (ii) $y(t) > 0$ for $t \in (0, \infty)$ and satisfies boundary condition $y(0) = 0$, y bounded on $[0, \infty)$;
- (iii) $y(t)$ satisfies each equality of (1.1).

Definition 1.2. The set \mathcal{F} is said to be quasi-equicontinuous in $[0, c]$ if for any $\varepsilon > 0$, there exists a $\delta > 0$ such that if $x \in \mathcal{F}$, $k \in \mathbb{Z}$, $t^*, t^{**} \in (t_{k-1}, t_k] \cap [0, c]$, and $|t^* - t^{**}| < \delta$, then $|x(t^*) - x(t^{**})| < \varepsilon$.

LEMMA 1.3 (compactness criterion [8]). *The set $\mathcal{F} \subset PC([0, c], \mathbb{R}^n)$ is relatively compact if and only if*

- (1) \mathcal{F} is bounded;
- (2) \mathcal{F} is quasi-equicontinuous in $[0, c]$.

2. Main results

THEOREM 2.1. *Let $g : [0, \infty) \times [0, \infty) \rightarrow \mathbb{R}$, $b = 0$, L^{-1} exist and is continuous.*

On the other hand, solving (8) is equivalent to finding a fixed point of

$$L^{-1}Ni : PC(I) \rightarrow PC(I) \quad (2.1)$$

with $i : PC^1(I) \rightarrow PC(I)$ the compact inclusion of $PC^1(I)$ in $PC(I)$. Now, Schauder's fixed point theorem guarantees the existence of at least a fixed point since $L^{-1}Ni$ is continuous and compact.

Next, prove that every solution u of (8) satisfies

$$\alpha(t) \leq u(t) \leq \beta(t) \quad \text{on } I. \quad (2.2)$$

By the definition of $p(t, x)$, $(0, \infty) \times [0, \infty) \rightarrow [0, \infty)$. Assume that the following hypothesis hold.

- (A₁) For any constant $H > 0$, there exists a function ψ_H continuous on $[0, \infty)$ and positive on $(0, \infty)$, and a constant γ , $0 \leq \gamma < 1$, with $g(t, u, v) \geq \psi_H(t)v^\gamma$ on $[0, \infty) \times [0, H]^2$.
 (A₂) There exist functions $p, r : [0, \infty) \rightarrow [0, \infty)$ such that

$$\begin{aligned} g(t, u, v) &\leq p(t)v + r(t) \quad \text{on } [0, \infty) \times [0, \infty)^2, \\ P_1 &= \int_0^\infty sp(s)ds < \infty, \quad R_1 = \int_0^\infty sr(s)ds < \infty, \\ P &= \int_0^\infty p(s)ds < 1, \quad R = \int_0^\infty r(s)ds < \infty. \end{aligned} \quad (2.3)$$

(A₃) $b_k \geq 0$, $a_k \geq -1$ and $\sum_{k=1}^\infty |a_k| \leq A < 1$.
 Then BVP (1.1) has at least one solution.

To prove Theorem 2.1, we need the following preliminary lemmas.

LEMMA 2.2. Let $e(t) \in C[0, \infty)$, $e(t) \geq 0$, $b_k \geq 0$, $x \in PC^1[0, \infty) \cap PC^2[0, \infty)$ be such that

$$\begin{aligned} x''(t) + e(t) &= 0, \quad t \in (0, b), \quad t \neq t_k, \\ \Delta x'(t_k) &= b_k x'(t_k), \end{aligned} \quad (2.4)$$

and $x(0) = 0$, $x'(b) = 0$. Then

$$\|x'\|_\infty \leq \int_0^b e(s)ds. \quad (2.5)$$

Proof. Since $-x''(t) = e(t)$, $x'(b) = 0$, then $x'(t) \geq 0$. Integrating from t to b we obtain

$$x'(t) = \int_t^b e(s)ds - \sum_{t < t_k < b} b_k x'(t_k) \leq \int_t^b e(s)ds \leq \int_0^b e(s)ds. \quad (2.6)$$

□

LEMMA 2.3. Let $g : [0, \infty) \times [0, \infty) \times [0, \infty) \rightarrow [0, \infty)$ and conditions (A₁)–(A₃) hold. Let n be a positive integer and consider the boundary value problem

$$\begin{aligned} y'' + g(t, y, y') &= 0, \quad 0 < t < n, \quad t \neq t_k, \\ \Delta y'(t_k) &= b_k y'(t_k), \quad \Delta y(t_k) = a_k y(t_k), \\ y(0) &= 0, \quad y'(n) = 0. \end{aligned} \quad (2.2_n)$$

Then (2.2_n) has at least one positive solution $y_n \in PC^1[0, n]$ and there is a constant $M > 0$

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independent of n such that

$$\left((1-\gamma) \int_t^n \prod_{t < t_k < s} (1+b_k)^{\gamma-1} \psi_M(s) ds \right)^{1/(1-\gamma)} \leq y'_n(t) \leq M, \quad t \in [0, n], \quad (2.7)$$

$$\int_0^t \prod_{s < t_k < t} (1+a_k) \left((1-\gamma) \int_s^n \prod_{s < t_k < \tau} (1+b_k)^{\gamma-1} \psi_M(\tau) d\tau \right)^{1/(1-\gamma)} ds \leq y_n(t) \leq M, \quad t \in [0, n]. \quad (2.8)$$

Proof. Let $n \in \mathbb{N}^+$ be fixed and $Y = X = PC^1[0, n]$. We first show that

$$\begin{aligned} y'' + g^*(t, y, y') &= 0, \quad 0 < t < n, \quad t \neq t_k, \\ \Delta y'(t_k) &= b_k y'(t_k), \quad \Delta y(t_k) = a_k y(t_k), \\ y(0) &= 0, \quad y'(n) = 0 \end{aligned} \quad (2.9)$$

has at least one solution, here

$$g^*(t, y, v) = \begin{cases} g(t, y, v), & y \geq 0, v \geq 0, \\ g(t, y, 0), & y \geq 0, v < 0, \\ g(t, 0, v), & y < 0, v \geq 0, \\ g(t, 0, 0), & y < 0, v < 0. \end{cases} \quad (2.10)$$

Define a linear operator $L_n : D(L_n) \subset X \rightarrow Y$ by setting

$$D(L_n) = \{x \in PC^2[0, n] : x(0) = x'(n) = 0\}, \quad (2.11)$$

and for $y \in D(L_n) : L_n y = (-y'', \Delta y'(t_k), \Delta y(t_k))$. We also define a nonlinear mapping $F : X \rightarrow Y$ by setting

$$(Fy)(t) = (g^*(t, y(t), y'(t)), b_k y'(t_k), a_k y(t_k)). \quad (2.12)$$

From the assumption of g , we see that F is a bounded mapping from X to Y . Next, it is easy to see that $L_n : D(L_n) \rightarrow Y$ is one-to-one mapping. Moreover, it follows easily using Lemma 1.3 that $(L_n)^{-1}F : X \rightarrow X$ is a compact mapping.

We note that $y \in PC^1[0, n]$ is a solution of (2.9) if and only if y is a fixed point of the equation

$$y = (L_n)^{-1}Fy. \quad (2.13)$$

We apply the Leray-Schauder continuation theorem to obtain the existence of a solution for $y = (L_n)^{-1}Fy$.

To do this, it suffices to verify that the set of all possible solutions of the family of equations

$$\begin{aligned} y'' + \lambda g^*(t, y, y') &= 0, \quad 0 < t < n, \quad t \neq t_k, \\ \Delta y'(t_k) &= \lambda b_k y'(t_k), \quad \Delta y(t_k) = \lambda a_k y(t_k), \\ y(0) &= y'(n) = 0 \end{aligned} \quad (2.5_\lambda)$$

is a priori bounded in $PC^1[0, n]$ by a constant independent of $0 < \lambda < 1$.

Let $y \in PC^1[0, n]$ be any solutions of (2.5_λ), then $y' \geq 0$ and $y \geq 0$ on $[0, n]$. Applying Lemma 2.2 and using (2.5_λ), we can get that

$$y'(t) \leq \int_0^n g^*(s, y(s), y'(s)) ds \leq \int_0^n p(s) y'(s) ds + \int_0^n r(s) ds \leq P \|y'\|_\infty + R, \quad (2.14)$$

so

$$\|y'\|_\infty \leq \frac{R}{1-P} := M_1. \quad (2.15)$$

From (2.5_λ) and $b_k \geq 0$, we have

$$y'(t) = \lambda \int_t^n g^*(s, y(s), y'(s)) ds - \lambda \sum_{t < t_k < n} b_k y'(t_k) \leq \int_t^n g^*(s, y(s), y'(s)) ds. \quad (2.16)$$

Integrate (2.16) from 0 to t to obtain

$$\begin{aligned} y(t) &\leq t \int_t^n g^*(s, y(s), y'(s)) ds + \int_0^t s g^*(s, y(s), y'(s)) ds + \lambda \sum_{0 < t_k < t} \Delta y(t_k) \\ &\leq \int_t^n s g^*(s, y(s), y'(s)) ds + \int_0^t s g^*(s, y(s), y'(s)) ds + \lambda \sum_{0 < t_k < t} a_k y(t_k) \\ &\leq \|y'\|_\infty \int_0^n s p(s) ds + \int_0^n s r(s) ds + \|y\|_\infty \sum_{0 < t_k < t} |a_k| \\ &\leq P_1 M_1 + R_1 + A \|y\|_\infty. \end{aligned} \quad (2.17)$$

Hence we have

$$\|y\|_\infty \leq \frac{P M_1 + R_1}{1-A} := M_2. \quad (2.18)$$

Let

$$M = \max \{M_1, M_2\}, \quad (2.19)$$

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it follows that

$$\|y\|_1 \leq M. \quad (2.20)$$

Note that M is independent of λ .

Therefore (2.20) implies that (2.5 $_\lambda$) has a solution y_n with $\|y_n\|_1 \leq M$. In fact,

$$0 \leq y_n(t) \leq M, \quad 0 \leq y'_n(t) \leq M \quad \text{for } t \in [0, n], \quad (2.21)$$

and y_n satisfies (2.2 $_n$).

Finally, it is easy to see from (2.19) that M is independent of $n \in \mathbb{N}^+$. Now (A $_1$) guarantees the existence of a function $\psi_M(t)$ continuous on $[0, \infty)$ and positive on $(0, \infty)$, a constant $\gamma \in [0, 1)$, with $g(t, y_n(t), y'_n(t)) \geq \psi_M(t)(y'_n(t))^\gamma$ for $(t, y_n(t), y'_n(t)) \in [0, n] \times [0, M]^2$.

From (2.2 $_n$) we have

$$-y''_n(t) \geq \psi_M(t)(y'_n(t))^\gamma, \quad (2.22)$$

integrate the above inequality from t to n to obtain

$$y'_n(t) \geq \left((1-\gamma) \int_t^n \prod_{t < t_k < s} (1+b_k)^{\gamma-1} \psi_M(s) ds \right)^{1/(1-\gamma)}, \quad t \in [0, n], \quad (2.23)$$

and so

$$y_n(t) \geq \int_0^t \prod_{s < t_k < \tau} (1+a_k) \left((1-\gamma) \int_s^n \prod_{s < t_k < \tau} (1+b_k)^{\gamma-1} \psi_M(\tau) d\tau \right)^{1/(1-\gamma)} ds, \quad t \in [0, n], \quad (2.24)$$

which completes the proof. \square

Proof of Theorem 2.1. From (2.2 $_n$) and (2.21), we know that

$$0 \leq -y''_n \leq \phi(t), \quad t \in [0, n], \quad (2.25)$$

where $\phi(t) := p(t)M + r(t)$, and M is given by (2.19). In addition, we have by $b_k \geq 0$ that

$$y'_n(t) \leq \int_t^n \phi(s) ds \leq \int_t^\infty \phi(s) ds \quad \text{for } t \in [0, n]. \quad (2.26)$$

To show that BVP (1.1) has a solution, we will apply the diagonalization argument. Let

$$u_n(t) = \begin{cases} y_n(t), & t \in [0, n], \\ y_n(n), & t \in [n, \infty). \end{cases} \quad (2.27)$$

Notice that $u_n \in PC^1[0, \infty)$ with

$$0 \leq u_n(t) \leq M, \quad 0 \leq u'_n(t) \leq M \quad \text{for } t \in [0, \infty). \quad (2.28)$$

From the definition of u_n , we get for $s_1, s_2 \in (t_k, t_{k+1}]$ that

$$|u'_n(s_1) - u'_n(s_2)| \leq \left| \int_{s_1}^{s_2} \phi(s) ds \right|. \quad (2.29)$$

In addition

$$u'_n(t) \leq \int_t^\infty \phi(s) ds \quad \text{for } t \in [0, \infty), \quad (2.30)$$

$$u_n(t) \geq \int_0^t \prod_{s < t_k < t} (1 + a_k) \left((1 - \gamma) \int_s^n \prod_{s < t_k < \tau} (1 + b_k)^{\gamma-1} \psi_M(\tau) d\tau \right)^{1/(1-\gamma)} ds, \quad t \in [0, n]. \quad (2.31)$$

In particular

$$\begin{aligned} u_n(t) &\geq \int_0^t \prod_{s < t_k < t} (1 + a_k) \left((1 - \gamma) \int_s^1 \prod_{s < t_k < \tau} (1 + b_k)^{\gamma-1} \psi_M(\tau) d\tau \right)^{1/(1-\gamma)} ds \\ &\equiv a_1(t), \quad t \in [0, 1]. \end{aligned} \quad (2.32)$$

Lemma 1.3 guarantees the existence of a subsequence N_1 of \mathbb{N}^+ and a function $z_1 \in PC^1[0, 1]$ with $u_n^{(j)}$ converging uniformly on $[0, 1]$ to $z_1^{(j)}$ as $n \rightarrow \infty$ through N_1 , here $j = 0, 1$. Also from (2.32), $z_1(t) \geq a_1(t)$ for $t \in [0, 1]$ (in particular, $z_1 > 0$ on $(0, 1]$).

Let $N_1^+ = N_1 \setminus \{1\}$, notice from (2.31) that

$$\begin{aligned} u_n(t) &\geq \int_0^t \prod_{s < t_k < t} (1 + a_k) \left((1 - \gamma) \int_s^2 \prod_{s < t_k < \tau} (1 + b_k)^{\gamma-1} \psi_M(\tau) d\tau \right)^{1/(1-\gamma)} ds \\ &\equiv a_2(t), \quad t \in [0, 2]. \end{aligned} \quad (2.33)$$

Lemma 1.3 guarantees the existence of a subsequence N_2 of N_1^+ and a function $z_2 \in PC^1[0, 2]$ with $u_n^{(j)}$ converging uniformly on $[0, 2]$ to $z_2^{(j)}$ as $n \rightarrow \infty$ through N_2 , here $j = 0, 1$. Also from (2.41), $z_2(t) \geq a_2(t)$ for $t \in [0, 2]$ (in particular, $z_2 > 0$ on $(0, 2]$). Note that $z_2 = z_1$ on $[0, 1]$, since $N_2 \subset N_1^+$. Let $N_2^+ = N_2 \setminus \{2\}$, proceed inductively to obtain for $k = 1, 2, \dots$, a subsequence N_k of N_{k-1}^+ and a function $z_k \in PC^1[0, k]$ with $u_n^{(j)}$ converging uniformly on $[0, k]$ to $z_k^{(j)}$ as $n \rightarrow \infty$ through N_k , here $j = 0, 1$. Also

$$\begin{aligned} z_k(t) &\geq a_k(t) \\ &\equiv \int_0^t \prod_{s < t_k < t} (1 + a_k) \left((1 - \gamma) \int_s^k \prod_{s < t_k < \tau} (1 + b_k)^{\gamma-1} \psi_M(\tau) d\tau \right)^{1/(1-\gamma)} ds, \quad t \in [0, k] \end{aligned} \quad (2.34)$$

(so in particular, $z_k > 0$ on $(0, k]$). Note that $z_k = z_{k-1}$ on $[0, k-1]$).

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Define a function y as follows: fix $t \in (0, \infty)$ and let $k \in N^+$ with $t < k$. Define $y(t) = z_k(t)$. Note that y is well defined and $y(t) = z_k(t) > 0$, we can do this for each $t \in (0, \infty)$ and so $y \in PC^1[0, \infty)$. In addition, $0 \leq y(t) \leq M$, $0 \leq y'(t) \leq M$, and

$$y'(t) \leq \int_t^\infty \phi(s) ds \quad \text{for } t \in [0, \infty). \quad (2.35)$$

Fix $x \in [0, \infty)$ and choose $k \geq x$, $k \in N^+$. Then for each $n \in N_k^+ = N_k \setminus \{k\}$, we have

$$\begin{aligned} y_n(x) &= y'_n(k)x + \int_0^x \int_s^k g(\tau, y_n(\tau), y'_n(\tau)) d\tau ds - \sum_{0 < t_i < k} b_i y'_n(t_i) x \\ &\quad + \sum_{0 < t_i \leq x} b_i y'_n(t_i) (x - t_i) + \sum_{0 < t_i < x} a_i y_n(t_i). \end{aligned} \quad (2.36)$$

Let $n \rightarrow \infty$ through N_k^+ to obtain

$$\begin{aligned} z_k(x) &= z'_k(k)x + \int_0^x \int_s^k g(\tau, z_k(\tau), z'_k(\tau)) d\tau ds \\ &\quad - \sum_{0 < t_i < k} b_i z'_k(t_i) x + \sum_{0 < t_i \leq x} b_i z'_k(t_i) (x - t_i) + \sum_{0 < t_i < x} a_i z_k(t_i). \end{aligned} \quad (2.37)$$

Thus

$$\begin{aligned} y(x) &= y'(k)x + \int_0^x \int_s^k g(\tau, y(\tau), y'(\tau)) d\tau ds \\ &\quad - \sum_{0 < t_i < k} b_i y'(t_i) x + \sum_{0 < t_i \leq x} b_i y'(t_i) (x - t_i) + \sum_{0 < t_i < x} a_i y(t_i). \end{aligned} \quad (2.38)$$

Consequently $y \in PC^2(0, \infty)$ with

$$\begin{aligned} y''(t) + g(t, y(t), y'(t)) &= 0, \quad 0 < t < \infty, \quad t \neq t_k, \\ \Delta y'(t_k) &= b_k y'(t_k), \quad \Delta y(t_k) = a_k y(t_k). \end{aligned} \quad (2.39)$$

Thus y is a solution of (1.1) with $y > 0$ on $(0, \infty)$. The proof is complete. \square

THEOREM 2.4. Let $g : [0, \infty) \times [0, \infty) \times [0, \infty) \rightarrow [0, \infty)$. Assume that (A_1) , (A_3) of Theorem 2.1 and the following condition hold.

(B₁) $g(t, x, v) \leq q(t)w(\max\{x, v\})$ on $[0, \infty) \times [0, \infty) \times [0, \infty)$ with $w > 0$ continuous and nondecreasing on $[0, \infty)$, $q(t) \in \mathbb{C}[0, \infty)$.

(B₂)

$$\begin{aligned} Q &= \int_0^\infty q(s) ds < \infty, \quad Q_1 = \int_0^\infty sq(s) ds < \infty, \\ \sup_{c \geq 0} \frac{c}{w(c)} &> T = \max \left\{ \frac{Q_1}{1 - A}, Q \right\}. \end{aligned} \quad (2.40)$$

Then BVP (1.1) has at least one positive solution.

Proof. Choose $M > 0$ with

$$\frac{M}{w(M)} > T. \quad (2.41)$$

We first show that (2.9) has at least one solution. To the end, we consider the operator

$$y = \lambda(L_n)^{-1}Fy, \quad \lambda \in (0, 1), \quad (2.42)$$

which is equivalent to (2.5 _{λ}). Let $y \in PC^1[0, n]$ be any solution of (2.5 _{λ}), then $y \geq 0$, $y' \geq 0$ on $[0, n]$. From (B₁) we have

$$-y''(t) \leq q(t)w(\|y\|_1) \quad \text{for } t \in [0, n]. \quad (2.43)$$

Integrate (2.43) from t to n to obtain

$$y'(t) \leq w(\|y\|_1) \int_t^n q(s)ds - \sum_{t < t_k < n} b_k y'(t_k) \leq w(\|y\|_1) \int_t^n q(s)ds \quad (2.44)$$

so

$$y'(t) \leq Qw(\|y\|_1). \quad (2.45)$$

Integrate (2.44) from 0 to t to obtain

$$y(t) \leq w(\|y\|_1) \int_0^t \int_s^n q(\tau)d\tau ds + \sum_{0 < t_k < t} a_k y(t_k) \leq w(\|y\|_1) \int_0^t sq(s)ds + A\|y\|_\infty. \quad (2.46)$$

Combine (2.45) and (2.46) to find

$$\|y\|_1 \leq Tw(\|y\|_1). \quad (2.47)$$

Now (2.41) together with (2.47) implies $\|y\|_1 \neq M$. Set

$$U = \{u \in PC^1[0, n] : \|u\|_1 < M\}, \quad K = E = PC^1[0, n]. \quad (2.48)$$

Now the nonlinear alternative of Leray-Schauder type [7] guarantees that $(L_n)^{-1}N$ has a fixed point, that is, (2.9) has a solution $y_n \in PC^1[0, n]$, and

$$0 \leq y_n \leq M, \quad 0 \leq y'_n \leq M. \quad (2.49)$$

The other proof is similar to the proof of Theorem 2.1, here we omit it. \square

3. Examples

Example 3.1. Consider the boundary value problem

$$\begin{aligned} y'' + \eta(y')^\beta e^{-t} + \mu e^{-t} &= 0, \quad 0 < t < \infty, \\ \Delta y'(t_k) &= \frac{1}{k} y'(t_k), \quad \Delta y(t_k) = \frac{2}{3k(k+1)} y(t_k), \quad k = 1, 2, \dots, \\ y(0) &= 0, \quad y \text{ bounded on } [0, \infty) \end{aligned} \quad (3.1)$$

with $\beta \in [0, 1)$, $\eta \in (0, 1)$, $\mu > 0$. Set $g(t, u, v) = \eta e^{-t} (y')^\beta + \mu e^{-t}$. Take $p(t) = \eta e^{-t}$, $r(t) = \mu e^{-t}$, then g satisfies (A_2) and $P = \eta < 1$. For each $H > 0$, take $\psi_H(t) = \eta e^{-t}$ and $\gamma = \beta$, then (A_1) is satisfied. Furthermore,

$$b_k = \frac{1}{k} > 0, \quad \sum_{k=1}^{\infty} |a_k| = \sum_{k=1}^{\infty} \frac{2}{3k(k+1)} = \frac{2}{3} < 1. \quad (3.2)$$

Therefore, Theorem 2.1 now guarantees that (3.1) has a solution $y \in PC^1[0, \infty)$ with $y > 0$ on $(0, \infty)$.

Example 3.2. Consider the boundary value problem

$$\begin{aligned} y'' + (y^\alpha + (y')^\beta) e^{-t} + \mu e^{-t} &= 0, \quad 0 < t < \infty, \\ \Delta y'(t_k) &= y'(t_k), \quad \Delta y(t_k) = \frac{1}{(k+1)^2} y(t_k), \quad k = 1, 2, \dots, \\ y(0) &= 0, \quad y \text{ bounded on } [0, \infty) \end{aligned} \quad (3.3)$$

with $\alpha \in [0, 1)$, $\beta \in [0, 1)$, $\mu > 0$. We will apply Theorem 2.4 with $q(t) = e^{-t}$, $w(s) = s^\alpha + s^\beta + \mu$. Clearly (A_1) , (A_3) , and (B_1) hold. Also,

$$\sup_{c \geq 0} \frac{c}{w(c)} = \sup_{c \geq 0} \frac{c}{c^\alpha + c^\beta + \mu} = \infty, \quad (3.4)$$

so (B_2) is true. Theorem 2.4 shows that (3.3) has a solution $y \in PC^1[0, \infty)$ with $y > 0$ on $(0, \infty)$.

Remark 3.3. We cannot apply the results of [12] even if (3.3) has no impulses, since [12, condition (2.3) of Theorem 2.1] is not satisfied.

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