Hindawi Publishing Corporation
Advances in Difference Equations
Volume 2010, Article ID 620459, 10 pages
doi:10.1155/2010/620459

Research Article

Monotone Iterative Technique for First-Order Nonlinear Periodic Boundary Value Problems on Time Scales

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Received 12 February 2010; Revised 17 May 2010; Accepted 26 June 2010

Academic Editor: Paul Eloe

We investigate the following nonlinear first-order periodic boundary value problem on time scales: $x^{\Delta}(t) + p(t)x(\sigma(t)) = f(t, x(t))$, $t \in [0, T]_{\mathbb{T}}$, $x(0) = x(\sigma(T))$. Some new existence criteria of positive solutions are established by using the monotone iterative technique.

1. Introduction

Recently, periodic boundary value problems (PBVPs for short) for dynamic equations on time scales have been studied by several authors by using the method of lower and upper solutions, fixed point theorems, and the theory of fixed point index. We refer the reader to [1–10] for some recent results.

In this paper we are interested in the existence of positive solutions for the following first-order PBVP on time scales:

$$\begin{gathered} x^{\Delta}(t) + p(t)x(\sigma(t)) = f(t, x(t)), \quad t \in [0, T]_{\mathbb{T}}, \\ x(0) = x(\sigma(T)), \end{gathered} \tag{1.1}$$

where σ will be defined in Section 2, $\mathbb{T}$ is a time scale, $T > 0$ is fixed and $0, T \in \mathbb{T}$. For each interval $\mathbf{I}$ of $\mathbb{R}$, we denote by $\mathbf{I}_{\mathbb{T}} = \mathbf{I} \cap \mathbb{T}$. By applying the monotone iterative technique, we obtain not only the existence of positive solution for the PBVP (1.1), but also give an iterative scheme, which approximates the solution. It is worth mentioning that the initial term of our iterative scheme is a constant function, which implies that the iterative scheme is significant and feasible. For abstract monotone iterative technique, see [11] and the references therein.

2. Some Results on Time Scales

Let us recall some basic definitions and relevant results of calculus on time scales [12–15].

Definition 2.1. For $t \in \mathbb{T}$, we define the forward jump operator $\sigma : \mathbb{T} \to \mathbb{T}$ by

$$\sigma(t) = \inf\{\tau \in \mathbb{T} : \tau > t\}, \tag{2.1}$$

while the backward jump operator $\rho : \mathbb{T} \to \mathbb{T}$ is defined by

$$\rho(t) = \sup\{\tau \in \mathbb{T} : \tau < t\}. \tag{2.2}$$

In this definition we put $\inf \emptyset = \sup \mathbb{T}$ and $\sup \emptyset = \inf \mathbb{T}$, where $\emptyset$ denotes the empty set. If $\sigma(t) > t$, we say that t is right scattered, while if $\rho(t) < t$, we say that t is left scattered. Also, if $t < \sup \mathbb{T}$ and $\sigma(t) = t$, then t is called right dense, and if $t > \inf \mathbb{T}$ and $\rho(t) = t$, then t is called left dense. We also need below the set $\mathbb{T}^k$ which is derived from the time scale $\mathbb{T}$ as follows. If $\mathbb{T}$ has a left-scattered maximum m, then $\mathbb{T}^k = \mathbb{T} - \{m\}$. Otherwise, $\mathbb{T}^k = \mathbb{T}$.

Definition 2.2. Assume that $x : \mathbb{T} \to \mathbb{R}$ is a function and let $t \in \mathbb{T}^k$. Then x is called differentiable at $t \in \mathbb{T}$ if there exists a $\theta \in \mathbb{R}$ such that, for any given $\epsilon > 0$, there is an open neighborhood U of t such that

$$|x(\sigma(t)) - x(s) - \theta|\sigma(t) - s|| \le \epsilon|\sigma(t) - s|, \quad s \in U. \tag{2.3}$$

In this case, θ is called the delta derivative of x at $t \in \mathbb{T}$ and we denote it by $\theta = x^{\Delta}(t)$. If $F^{\Delta}(t) = f(t)$, then we define the integral by

$$\int_a^t f(s)\Delta s = F(t) - F(a). \tag{2.4}$$

Definition 2.3. A function $f : \mathbb{T} \to \mathbb{R}$ is called rd-continuous provided that it is continuous at right-dense points in $\mathbb{T}$ and its left-sided limits exist at left-dense points in $\mathbb{T}$. The set of rd-continuous functions $f : \mathbb{T} \to \mathbb{R}$ will be denoted by C_{rd}.

Lemma 2.4 (see [13]). *If $f \in C_{\mathrm{rd}}$ and $t \in \mathbb{T}^k$, then*

$$\int_t^{\sigma(t)} f(s)\Delta s = \mu(t)f(t), \tag{2.5}$$

where $\mu(t) = \sigma(t) - t$ is the graininess function.

Lemma 2.5 (see [13]). *If $f^{\Delta} > 0$, then f is increasing.*

Definition 2.6. For $h > 0$, we define the Hilger complex numbers as

$$\mathbb{C}_h = \left\{ z \in \mathbb{C} : z \neq -\frac{1}{h} \right\}, \tag{2.6}$$

and for $h = 0$, let $\mathbb{C}_0 = \mathbb{C}$.

Definition 2.7. For $h > 0$, let $\mathbb{Z}_h$ be the strip

$$\mathbb{Z}_h = \left\{ z \in \mathbb{C} : -\frac{\pi}{h} < Im(z) \leq \frac{\pi}{h} \right\}, \tag{2.7}$$

and for $h = 0$, let $\mathbb{Z}_0 = \mathbb{C}$.

Definition 2.8. For $h > 0$, we define the cylinder transformation $\xi_h : \mathbb{C}_h \to \mathbb{Z}_h$ by

$$\xi_h(z) = \frac{1}{h}\operatorname{Log}(1 + zh), \tag{2.8}$$

where Log is the principal logarithm function. For $h = 0$, we define $\xi_0(z) = z$ for all $z \in \mathbb{C}$.

Definition 2.9. A function $p : \mathbb{T} \to \mathbb{R}$ is regressive provided that

$$1 + \mu(t)p(t) \neq 0, \quad \forall t \in \mathbb{T}^k. \tag{2.9}$$

The set of all regressive and rd-continuous functions will be denoted by $\mathcal{R}$.

Definition 2.10. We define the set $\mathcal{R}^+$ of all positively regressive elements of $\mathcal{R}$ by

$$\mathcal{R}^+ = \{ p \in \mathcal{R} : 1 + \mu(t)p(t) > 0,\ \forall t \in \mathbb{T} \}. \tag{2.10}$$

Definition 2.11. If $p \in \mathcal{R}$, then the generalized exponential function is given by

$$e_p(t, s) = \exp\left(\int_s^t \xi_{\mu(\tau)}(p(\tau))\Delta\tau \right), \quad \text{for } s, t \in \mathbb{T}, \tag{2.11}$$

where the cylinder transformation $\xi_h(z)$ is defined as in Definition 2.8.

Lemma 2.12 (see [13]). *If $p \in \mathcal{R}$, then*

(i) $e_p(t, t) \equiv 1$,

(ii) $e_p(t, s) = 1/e_p(s, t)$,

(iii) $e_p(t, u)e_p(u, s) = e_p(t, s)$,

(iv) $e_p^{\Delta}(t, t_0) = p(t)e_p(t, t_0)$, *for* $t \in \mathbb{T}^k$ *and* $t_0 \in \mathbb{T}$.

Lemma 2.13 (see [13]). *If $p \in \mathcal{R}^+$ and $t_0 \in \mathbb{T}$, then*

$$e_p(t, t_0) > 0, \quad \forall t \in \mathbb{T}. \tag{2.12}$$

3. Main Results

For the forthcoming analysis, we assume that the following two conditions are satisfied.

(H1) $p : [0, T]_{\mathbb{T}} \to (0, +\infty)$ is rd-continuous, which implies that $p \in \mathcal{R}^+$.

(H2) $f : [0, T]_{\mathbb{T}} \times [0, +\infty) \to [0, +\infty)$ is continuous and $f(t, x)$ is nondecreasing on x.

If we denote that

$$A = \frac{1}{e_p(\sigma(T), 0) - 1}, \qquad \delta = \left(\frac{A}{1 + A}\right)^2, \tag{3.1}$$

then we may claim that $A > 0$, which implies that $0 < \delta < 1$.

In fact, in view of (H1) and Lemmas 2.12 and 2.13, we have

$$e_p^{\Delta}(t, 0) = p(t)e_p(t, 0) > 0, \quad t \in [0, T]_{\mathbb{T}}, \tag{3.2}$$

which together with Lemma 2.5 shows that $e_p(t, 0)$ is increasing on $[0, \sigma(T)]_{\mathbb{T}}$. And so,

$$e_p(\sigma(T), 0) > e_p(0, 0) = 1. \tag{3.3}$$

This indicates that $A > 0$.

Let

$$\mathbb{E} = \{x \mid x : [0, \sigma(T)]_{\mathbb{T}} \longrightarrow \mathbb{R} \text{ is continuous}\} \tag{3.4}$$

be equipped with the norm $\|x\| = \max_{t \in [0, \sigma(T)]_{\mathbb{T}}} |x(t)|$. Then $\mathbb{E}$ is a Banach space.

First, we define two cones K and P in $\mathbb{E}$ as follows:

$$\begin{gathered} K = \{x \in \mathbb{E} \mid x(t) \geq 0,\ t \in [0, \sigma(T)]_{\mathbb{T}}\}, \\ P = \{x \in K \mid x(t) \geq \delta \|x\|,\ t \in [0, \sigma(T)]_{\mathbb{T}}\}, \end{gathered} \tag{3.5}$$

and then we define an operator $\Phi : K \to K$:

$$(\Phi x)(t) = \frac{1}{e_p(t, 0)} \left[\int_0^t e_p(s, 0) f(s, x(s)) \Delta s + A \int_0^{\sigma(T)} e_p(s, 0) f(s, x(s)) \Delta s \right], \quad t \in [0, \sigma(T)]_{\mathbb{T}}. \tag{3.6}$$

It is obvious that fixed points of Φ are solutions of the PBVP (1.1).

Since $f(t, x)$ is nondecreasing on x, we have the following lemma.

Lemma 3.1. *$\Phi : K \to K$ is nondecreasing.*

Lemma 3.2. *$\Phi : P \to P$ is completely continuous.*

Proof. Suppose that $x \in P$. Then

$$\begin{aligned}
0 \le (\Phi x)(t) &= \frac{1}{e_p(t,0)}\left[\int_0^t e_p(s,0)f(s,x(s))\Delta s + A\int_0^{\sigma(T)} e_p(s,0)f(s,x(s))\Delta s\right] \\
&\le \int_0^t e_p(s,0)f(s,x(s))\Delta s + A\int_0^{\sigma(T)} e_p(s,0)f(s,x(s))\Delta s \\
&\le (1+A)\int_0^{\sigma(T)} e_p(s,0)f(s,x(s))\Delta s, \quad t \in [0,\sigma(T)]_{\mathbb{T}},
\end{aligned} \tag{3.7}$$

so,

$$\|\Phi x\| \le (1+A)\int_0^{\sigma(T)} e_p(s,0)f(s,x(s))\Delta s. \tag{3.8}$$

Therefore,

$$\begin{aligned}
(\Phi x)(t) &= \frac{1}{e_p(t,0)}\left[\int_0^t e_p(s,0)f(s,x(s))\Delta s + A\int_0^{\sigma(T)} e_p(s,0)f(s,x(s))\Delta s\right] \\
&\ge \frac{A}{e_p(\sigma(T),0)}\int_0^{\sigma(T)} e_p(s,0)f(s,x(s))\Delta s \\
&\ge \delta\|\Phi x\|, \quad t \in [0,\sigma(T)]_{\mathbb{T}}.
\end{aligned} \tag{3.9}$$

This shows that $\Phi : P \to P$. Furthermore, with similar arguments as in [7], we can prove that $\Phi : P \to P$ is completely continuous by Arzela-Ascoli theorem. □

Theorem 3.3. *Assume that there exist two positive numbers $R_1 < R_2$ such that*

$$\inf_{t\in[0,T]_{\mathbb{T}}} f(t,\delta R_1) \ge \frac{(1+A)R_1}{A^2\sigma(T)}, \qquad \sup_{t\in[0,T]_{\mathbb{T}}} f(t,R_2) \le \frac{AR_2}{(1+A)^2\sigma(T)}. \tag{3.10}$$

Then the PBVP (1.1) *has positive solutions x^* and y^*, which may coincide with*

$$\begin{aligned}
&\delta R_1 \le x^*(t) \le R_2, \quad \text{for } t \in [0,\sigma(T)]_{\mathbb{T}}, \quad \lim_{n\to+\infty}\Phi^n x_0 = x^*, \\
&\delta R_1 \le y^*(t) \le R_2, \quad \text{for } t \in [0,\sigma(T)]_{\mathbb{T}}, \quad \lim_{n\to+\infty}\Phi^n y_0 = y^*,
\end{aligned} \tag{3.11}$$

where $x_0(t) \equiv R_2$ and $y_0(t) \equiv R_1$ for $t \in [0,\sigma(T)]_{\mathbb{T}}$.

Proof. First, we define

$$P_{[R_1,R_2]} = \{x \in P : R_1 \le \|x\| \le R_2\}. \tag{3.12}$$

Then we may assert that

$$\Phi\left(P_{[R_1,R_2]}\right) \subset P_{[R_1,R_2]}. \tag{3.13}$$

In fact, if $x \in P_{[R_1,R_2]}$, then

$$\delta R_1 \le \delta\|x\| \le x(t) \le \|x\| \le R_2, \quad \text{for } t \in [0,\sigma(T)]_{\mathbb{T}}, \tag{3.14}$$

which together with (H2) and (3.10) implies that

$$\begin{aligned}
(\Phi x)(t) &= \frac{1}{e_p(t,0)}\left[\int_0^t e_p(s,0)f(s,x(s))\Delta s + A\int_0^{\sigma(T)} e_p(s,0)f(s,x(s))\Delta s\right] \\
&\ge \frac{A}{e_p(\sigma(T),0)}\int_0^{\sigma(T)} e_p(s,0)f(s,x(s))\Delta s \\
&\ge \frac{A}{e_p(\sigma(T),0)}\int_0^{\sigma(T)} f(s,x(s))\Delta s \\
&\ge \frac{A}{e_p(\sigma(T),0)}\int_0^{\sigma(T)} f(s,\delta R_1)\Delta s \\
&\ge R_1, \quad t \in [0,\sigma(T)]_{\mathbb{T}}, \\
(\Phi x)(t) &= \frac{1}{e_p(t,0)}\left[\int_0^t e_p(s,0)f(s,x(s))\Delta s + A\int_0^{\sigma(T)} e_p(s,0)f(s,x(s))\Delta s\right] \\
&\le \int_0^t e_p(s,0)f(s,x(s))\Delta s + A\int_0^{\sigma(T)} e_p(s,0)f(s,x(s))\Delta s \\
&\le (1+A)\int_0^{\sigma(T)} e_p(s,0)f(s,x(s))\Delta s \\
&\le (1+A)e_p(\sigma(T),0)\int_0^{\sigma(T)} f(s,x(s))\Delta s \\
&\le (1+A)e_p(\sigma(T),0)\int_0^{\sigma(T)} f(s,R_2)\Delta s \\
&\le R_2, \quad t \in [0,\sigma(T)]_{\mathbb{T}},
\end{aligned} \tag{3.15}$$

which shows that

$$\Phi\left(P_{[R_1,R_2]}\right) \subset P_{[R_1,R_2]}. \tag{3.16}$$

Now, if we denote that $x_0(t) \equiv R_2$ for $t \in [0, \sigma(T)]_{\mathbb{T}}$, then $x_0 \in P_{[R_1,R_2]}$. Let

$$x_{n+1} = \Phi x_n, \quad n = 0, 1, 2, \ldots. \tag{3.17}$$

In view of $\Phi(P_{[R_1,R_2]}) \subset P_{[R_1,R_2]}$, we have $x_n \in P_{[R_1,R_2]}$, $n = 0, 1, 2, \ldots$. Since the set $\{x_n\}_{n=0}^{\infty}$ is bounded and the operator Φ is compact, we know that the set $\{x_n\}_{n=1}^{\infty}$ is relatively compact, which implies that there exists a subsequence $\{x_{n_k}\}_{k=1}^{\infty} \subset \{x_n\}_{n=1}^{\infty}$ such that

$$\lim_{k \to +\infty} x_{n_k} = x^* \in P_{[R_1,R_2]}. \tag{3.18}$$

Moreover, since

$$0 \le x_1(t) \le \|x_1\| \le R_2 = x_0(t), \quad \text{for } t \in [0, \sigma(T)]_{\mathbb{T}}, \tag{3.19}$$

it follows from Lemma 3.1 that $\Phi x_1 \le \Phi x_0$; that is, $x_2 \le x_1$. By induction, it is easy to know that

$$x_{n+1} \le x_n, \quad n = 1, 2, \ldots, \tag{3.20}$$

which together with (3.18) implies that

$$\lim_{n \to +\infty} x_n = x^* \in P_{[R_1,R_2]}. \tag{3.21}$$

Since Φ is continuous, it follows from (3.17) and (3.21) that

$$\Phi x^* = x^*, \tag{3.22}$$

which shows that x^* is a solution of the PBVP (1.1). Furthermore, we get from $x^* \in P_{[R_1,R_2]}$ that

$$\delta R_1 \le \delta \|x^*\| \le x^*(t) \le \|x^*\| \le R_2, \quad \text{for } t \in [0, \sigma(T)]_{\mathbb{T}}. \tag{3.23}$$

On the other hand, if we denote that $y_0(t) \equiv R_1$ for $t \in [0, \sigma(T)]_{\mathbb{T}}$ and that $y_{n+1} = \Phi y_n, n = 0, 1, 2, \ldots$, then we can obtain similarly that $y_n \in P_{[R_1,R_2]}$, $n = 0, 1, 2, \ldots$, and there exists a subsequence $\{y_{n_k}\}_{k=1}^{\infty} \subset \{y_n\}_{n=1}^{\infty}$ such that

$$\lim_{k \to +\infty} y_{n_k} = y^* \in P_{[R_1,R_2]}. \tag{3.24}$$

Moreover, since

$$\begin{aligned} y_1(t) &= (\Phi y_0)(t) \\ &= \frac{1}{e_p(t,0)}\left[\int_0^t e_p(s,0)f(s,y_0(s))\Delta s + A\int_0^{\sigma(T)} e_p(s,0)f(s,y_0(s))\Delta s\right] \\ &\geq \frac{A}{e_p(\sigma(T),0)}\int_0^{\sigma(T)} e_p(s,0)f(s,y_0(s))\Delta s \\ &\geq \frac{A}{e_p(\sigma(T),0)}\int_0^{\sigma(T)} f(s,\delta R_1)\Delta s \\ &\geq R_1 = y_0(t), \quad t\in[0,\sigma(T)]_{\mathbb{T}}, \end{aligned} \tag{3.25}$$

it is also easy to know that

$$y_n \leq y_{n+1}, \quad n = 1,2,\ldots. \tag{3.26}$$

With the similar arguments as above, we can prove that y^* is a solution of the PBVP (1.1) and satisfies

$$\delta R_1 \leq y^*(t) \leq R_2, \quad \text{for } t\in[0,\sigma(T)]_{\mathbb{T}}. \tag{3.27}$$

□

Corollary 3.4. *If the following conditions are fulfilled:*

$$\lim_{x\to 0^+}\inf_{t\in[0,T]_{\mathbb{T}}}\frac{f(t,x)}{x} = +\infty, \qquad \lim_{x\to +\infty}\sup_{t\in[0,T]_{\mathbb{T}}}\frac{f(t,x)}{x} = 0, \tag{3.28}$$

then there exist two positive numbers $R_1 < R_2$ such that (3.10) *is satisfied, which implies that the PBVP* (1.1) *has positive solutions x^* and y^*, which may coincide with*

$$\begin{aligned} &\delta R_1 \leq x^*(t) \leq R_2, \quad \text{for } t\in[0,\sigma(T)]_{\mathbb{T}}, \quad \lim_{n\to+\infty}\Phi^n x_0 = x^*, \\ &\delta R_1 \leq y^*(t) \leq R_2, \quad \text{for } t\in[0,\sigma(T)]_{\mathbb{T}}, \quad \lim_{n\to+\infty}\Phi^n y_0 = y^*, \end{aligned} \tag{3.29}$$

where $x_0(t)\equiv R_2$ and $y_0(t)\equiv R_1$ for $t\in[0,\sigma(T)]_{\mathbb{T}}$.

Example 3.5. Let $\mathbb{T} = [0,1]\cup[2,3]$. We consider the following PBVP on $\mathbb{T}$:

$$\begin{aligned} x^{\Delta}(t) + x(\sigma(t)) &= (t+1)\sqrt{x(t)}, \quad t\in[0,3]_{\mathbb{T}}, \\ x(0) &= x(3). \end{aligned} \tag{3.30}$$

Since $p(t) \equiv 1, \mathbb{T} = [0,1] \cup [2,3]$ and $T = 3$, we can obtain that

$$\sigma(T) = 3, \qquad A = \frac{1}{2e^2 - 1}, \qquad \delta = \frac{1}{4e^4}. \tag{3.31}$$

Thus, if we choose $R_1 = 9/16e^8(2e^2-1)^2$ and $R_2 = 2304e^8/(2e^2-1)^2$, then all the conditions of Theorem 3.3 are fulfilled. So, the PBVP (3.30) has positive solutions x^* and y^*, which may coincide with

$$\begin{aligned} &\frac{9}{64e^{12}(2e^2-1)^2} \le x^*(t) \le \frac{2304e^8}{(2e^2-1)^2}, \quad \text{for } t \in [0,3]_{\mathbb{T}}, \quad \lim_{n\to+\infty} \Phi^n x_0 = x^*, \\ &\frac{9}{64e^{12}(2e^2-1)^2} \le y^*(t) \le \frac{2304e^8}{(2e^2-1)^2}, \quad \text{for } t \in [0,3]_{\mathbb{T}}, \quad \lim_{n\to+\infty} \Phi^n y_0 = y^*, \end{aligned} \tag{3.32}$$

where $x_0(t) \equiv 2304e^8/(2e^2-1)^2$ and $y_0(t) \equiv 9/16e^8(2e^2-1)^2$ for $t \in [0,3]_{\mathbb{T}}$.

Acknowledgment

This work was supported by the National Natural Science Foundation of China (10801068).

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