MAXIMUM NORM ANALYSIS OF AN OVERLAPPING NONMATCHING GRIDS METHOD FOR THE OBSTACLE PROBLEM

M. BOULBRACHENE AND S. SAADI

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We provide a maximum norm analysis of an overlapping Schwarz method on nonmatching grids for second-order elliptic obstacle problem. We consider a domain which is the union of two overlapping subdomains where each subdomain has its own independently generated grid. The grid points on the subdomain boundaries need not match the grid points from the other subdomain. Under a discrete maximum principle, we show that the discretization on each subdomain converges quasi-optimally in the L^∞ norm.

1. Introduction

The Schwarz alternating method can be used to solve elliptic boundary value problems on domains which consists of two or more overlapping subdomains. The solution is approximated by an infinite sequence of functions which results from solving a sequence of elliptic boundary value problems in each of the subdomain.

Extensive analysis of Schwarz alternating method for continuous obstacle problem can be found in [8, 9]. For convergence of discrete Schwarz algorithms of either additive or multiplicative types, see for example, [1, 6, 7, 11].

In this paper, we are interested in the error analysis in the maximum norm for the obstacle problem in the context of overlapping nonmatching grids: we consider a domain Ω which is the union of two overlapping subdomains where each subdomain has its own triangulation. This kind of discretizations is very interesting as they can be applied to solving many practical problems which cannot be handled by global discretizations. They are earning particular attention of computational experts and engineers as they allow the choice of different mesh sizes and different orders of approximate polynomials in different subdomains according to the different properties of the solution and different requirements of the practical problems.

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To prove the main result, we develop an approach which combines a geometrical convergence result due to Lions [9] and a lemma which consists of estimating the error in the L^∞ norm between the continuous and discrete Schwarz iterates. The convergence order is then derived making use of standard finite element L^∞-error estimate for elliptic variational inequalities.

Quite a few works on maximum error analysis of overlapping nonmatching grid methods are known in the literature (cf., e.g., [2, 3, 10]). However, to the best of our knowledge, this is the first paper that provides an L^∞-error analysis for overlapping nonmatching grids for variational inequalities.

Now we give an outline of the paper. In Section 2. we state the continuous alternating Schwarz sequences for the obstacle problem, and define their respective finite element counterparts in the context of nonmatching overlapping grids. Section 3. is devoted to the L^∞-error analysis of the method.

2. The Schwarz method for the obstacle problem

We begin by laying down some definitions and classical results related to elliptic variational inequalities.

2.1. Elliptic obstacle problem. Let Ω be a convex domain in $\mathbb{R}^2$ with sufficiently smooth boundary $\partial\Omega$. We consider the bilinear form

$$a(u,v) = \int_\Omega (\nabla u \cdot \nabla v)\,dx, \tag{2.1}$$

the linear form

$$(f,v) = \int_\Omega f(x) \cdot v(x)\,dx, \tag{2.2}$$

the right-hand side

$$f \in L^\infty(\Omega), \tag{2.3}$$

the obstacle

$$\psi \in W^{2,\infty}(\Omega) \quad \text{such that } \psi \geq 0 \text{ on } \partial\Omega, \tag{2.4}$$

and the nonempty convex set

$$K_g = \{v \in H^1(\Omega) : v = g \text{ on } \partial\Omega,\ v \leq \psi \text{ on } \Omega\}, \tag{2.5}$$

where g is a regular function defined on $\partial\Omega$.

We consider the obstacle problem: find $u \in K_g$ such that

$$a(u, v-u) \geq (f, v-u), \quad \forall v \in K_g. \tag{2.6}$$

Let V_h be the space of finite elements consisting of continuous piecewise linear functions. The discrete counterpart of (2.6) consists of finding $u_h \in K_{gh}$ such that

$$a(u_h, v - u_h) \geq (f, v - u_h) \quad \forall v \in K_{gh}, \tag{2.7}$$

where

$$K_{gh} = \{v \in V_h : v = \pi_h g \text{ on } \partial\Omega,\ v \leq r_h \psi \text{ on } \Omega\} \tag{2.8}$$

π_h is an interpolation operator on $\partial\Omega$, and r_h is the usual finite element restriction operator on Ω.

The lemma below establishes a monotonicity property of the solution of (2.6) with respect to the obstacle and the boundary condition.

Lemma 2.1. *Let (ψ, g); $(\tilde{\psi}, \tilde{g})$ be a pair of data, and $u = \sigma(\psi, g)$; $\tilde{u} = \sigma(\tilde{\psi}, \tilde{g})$ the corresponding solutions to (2.6). If $\psi \geq \tilde{\psi}$ and $g \geq \tilde{g}$, then $\sigma(\psi, g) \geq \sigma(\tilde{\psi}, \tilde{g})$.*

Proof. Let $v = \min(0, u - \tilde{u})$. In the region where v is negative ($v < 0$), we have

$$u < \tilde{u} \leq \tilde{\psi} \leq \psi \tag{2.9}$$

which means that the obstacle is not active for u. So, for that v, we have

$$a(u, v) = (f, v), \tag{2.10}$$

$$\tilde{u} + v \leq \tilde{\psi} \tag{2.11}$$

so

$$a(\tilde{u}, v) \geq (f, v). \tag{2.12}$$

Subtracting (2.10) and (2.12) from each other, we obtain

$$a(\tilde{u} - u, v) \geq 0. \tag{2.13}$$

But,

$$a(v, v) = a(u - \tilde{u}, v) = -a(\tilde{u} - u, v) \leq 0 \tag{2.14}$$

so

$$v = 0 \tag{2.15}$$

and consequently,

$$u \geq \tilde{u} \tag{2.16}$$

which completes the proof. □

The proof for the discrete case is similar.

Proposition 2.2. *Under the notations and conditions of the preceding lemma, we have*

$$\|u-\tilde{u}\|_{L^\infty(\Omega)} \leq \|\psi-\tilde{\psi}\|_{L^\infty(\Omega)} + \|g-\tilde{g}\|_{L^\infty(\partial\Omega)}. \tag{2.17}$$

Proof. Setting

$$\Phi = \|\psi-\tilde{\psi}\|_{L^\infty(\Omega)} + \|g-\tilde{g}\|_{L^\infty(\partial\Omega)} \tag{2.18}$$

we have

$$\begin{aligned}\psi &\leq \tilde{\psi}+\psi-\tilde{\psi} \leq \tilde{\psi}+|\psi-\tilde{\psi}| \leq \tilde{\psi}+\|\psi-\tilde{\psi}\|_{L^\infty(\Omega)}\\ &\leq \tilde{\psi}+\|\psi-\tilde{\psi}\|_{L^\infty(\Omega)}+\|g-\tilde{g}\|_{L^\infty(\partial\Omega)}\end{aligned} \tag{2.19}$$

hence

$$\psi \leq \tilde{\psi}+\Phi. \tag{2.20}$$

On the other hand, we have

$$\begin{aligned}g &\leq \tilde{g}+g-\tilde{g} \leq \tilde{g}+|g-\tilde{g}| \leq \tilde{g}+\|g-\tilde{g}\|_{L^\infty(\partial\Omega)}\\ &\leq \tilde{g}+\|g-\tilde{g}\|_{L^\infty(\partial\Omega)}+\|\psi-\tilde{\psi}\|_{L^\infty(\Omega)}\end{aligned} \tag{2.21}$$

so

$$g \leq \tilde{g}+\Phi. \tag{2.22}$$

Now, making use of Lemma 2.1, we obtain

$$\sigma(\psi,g) \leq \sigma(\tilde{\psi}+\Phi,\tilde{g}+\Phi) = \sigma(\tilde{\psi},\tilde{g})+\Phi \tag{2.23}$$

or

$$\sigma(\psi,g)-\sigma(\tilde{\psi},\tilde{g}) \leq \Phi. \tag{2.24}$$

Similarly, interchanging the roles of the couples (ψ,g) and $(\tilde{\psi},\tilde{g})$, we obtain

$$\sigma(\tilde{\psi},\tilde{g})-\sigma(\psi,g) \leq \Phi. \tag{2.25}$$

The proof for the discrete case is similar. □

Remark 2.3. If $\psi=\tilde{\psi}$, then (2.17) becomes

$$\|u-\tilde{u}\|_{L^\infty(\Omega)} \leq \|g-\tilde{g}\|_{L^\infty(\partial\Omega)}. \tag{2.26}$$

Theorem 2.4 (cf. [5]). *Under conditions (2.3) and (2.4), there exists a constant C independent of h such that*

$$\|u-u_h\|_{L^\infty(\Omega)} \leq Ch^2|\ln h|^2. \tag{2.27}$$

2.2. The continuous Schwarz sequences. Consider the model obstacle problem: find $u \in K_0$ $(g = 0)$ such that

$$a(u, v-u) \geq (f, v-u) \quad \forall v \in K_0. \tag{2.28}$$

We decompose Ω into two overlapping polygonal subdomains Ω_1 and Ω_2 such that

$$\Omega = \Omega_1 \cup \Omega_2 \tag{2.29}$$

and u satisfies the local regularity condition

$$u/\Omega_i \in W^{2,p}(\Omega_i); \quad 2 \leq p < \infty. \tag{2.30}$$

We denote by $\partial\Omega_i$ the boundary of Ω_i, and $\Gamma_i = \partial\Omega_i \cap \Omega_j$. The intersection of $\overline{\Gamma}_i$ and $\overline{\Gamma}_j$; $i \neq j$ is assumed to be empty.

Choosing $u^0 = \psi$, we respectively define the alternating Schwarz sequences (u_1^{n+1}) on Ω_1 such that $u_1^{n+1} \in K$ solves

$$\begin{gathered} a_1(u_1^{n+1}, v - u_1^{n+1}) \geq (f_1, v - u_1^n) \quad \forall v \in K, \\ u_1^{n+1} = u_2^n \quad \text{on } \Gamma_1,\ v = u_2^n \text{ on } \Gamma_1 \end{gathered} \tag{2.31}$$

and (u_2^{n+1}) on Ω_2 such that $u_2^{n+1} \in K$ solves

$$\begin{gathered} a_2(u_2^{n+1}, v - u_2^{n+1}) \geq (f_2, v - u_2^{n+1}) \quad \forall v \in K, \\ u_2^{n+1} = u_1^{n+1} \quad \text{on } \Gamma_2;\ v = u_1^{n+1} \text{ on } \Gamma_2, \end{gathered} \tag{2.32}$$

where

$$f_i = f/_{\Omega_i}, \qquad a_i(u,v) = \int_{\Omega_i} (\nabla u \nabla v)\,dx. \tag{2.33}$$

The following geometrical convergence is due to Lions [9].

2.3. Geometrical convergence.

Theorem 2.5 (cf. [9]). *The sequences (u_1^{n+1}); (u_2^{n+1}); $n \geq 0$ produced by the Schwarz alternating method converge geometrically to the solution u of the obstacle problem (2.28). More precisely, there exist two constants $k_1, k_2 \in (0,1)$ such that for all $n \geq 0$,*

$$\begin{gathered} \|u_1 - u_1^{n+1}\|_{L^\infty(\Omega_1)} \leq k_1^n k_2^n \|u^0 - u\|_{L^\infty(\Gamma_1)}, \\ \|u_2 - u_2^{n+1}\|_{L^\infty(\Omega_2)} \leq k_1^{n+1} k_2^n \|u^0 - u\|_{L^\infty(\Gamma_2)}, \end{gathered} \tag{2.34}$$

where $u_i = u/\Omega_i$, $i = 1,2$.

2.4. The discretization. For $i = 1,2$, let τ^{h_i} be a standard regular and quasi-uniform finite element triangulation in Ω_i; h_i, being the meshsize. We assume that the two triangulations are mutually independent on $\Omega_1 \cap \Omega_2$ in the sense that a triangle belonging to one triangulation does not necessarily belong to the other.

Let $V_{h_i} = V_{h_i}(\Omega_i)$ be the space of continuous piecewise linear functions on τ^{h_i} which vanish on $\partial\Omega \cap \partial\Omega_i$. For $w \in C(\bar{\Gamma}_i)$ we define

$$V_{h_i}^{(w)} = \{v \in V_{h_i} : v = 0 \text{ on } \partial\Omega_i \cap \partial\Omega;\ v = \pi_{h_i}(w) \text{ on } \Gamma_i\}, \tag{2.35}$$

where π_{h_i} denotes the interpolation operator on Γ_i.

We also assume that the respective matrices resulting from the discretizations of problems (2.31) and (2.32), are M-matrices. (see [4]).

We now define the discrete counterparts of the continuous Schwarz sequences defined in (2.31) and (2.32), respectively by: $u_{1h}^{n+1} \in V_{h_1}^{(u_{2h}^n)}$ such that

$$\begin{gathered} a_1(u_{1h}^{n+1}, v - u_{1h}^{n+1}) \geq (f_1, v - u_{1h}^{n+1}) \quad \forall v \in V_{h_1}^{(u_{2h}^n)}, \\ u_{1h}^{n+1} \leq r_h, \qquad v \leq r_h\psi \end{gathered} \tag{2.36}$$

and $u_{2h}^{n+1} \in V_{h_2}^{(u_{1h}^{n+1})}$ such that

$$\begin{gathered} a_2(u_{2h}^{n+1}, v - u_{2h}^{n+1}) \geq (f_2, v - u_{2h}^{n+1}) \quad \forall v \in V_{h_2}^{(u_{1h}^{n+1})} \\ u_{2h}^{n+1} \leq r_h, \qquad v \leq r_h\psi. \end{gathered} \tag{2.37}$$

Remark 2.6. As the two meshes τ^{h_1} and τ^{h_2} are independent over the overlapping subdomains, it is impossible to formulate a global approximate problem which would be the direct discrete counterpart of problem (2.28).

3. L^∞-error analysis

This section is devoted to the proof of the main result of the present paper. To that end we begin by introducing two discrete auxiliary sequences and prove a fundamental lemma.

3.1. Definition of two auxiliary sequences. For $\omega_{ih}^0 = u_{ih}^0 = r_h\psi$; $i = 1,2$, we define the sequences (ω_{1h}^{n+1}) such that $\omega_{1h}^{n+1} \in V_{h_1}^{(u_2^n)}$ solves

$$\begin{gathered} a_1(\omega_{1h}^{n+1}, v - \omega_{1h}^{n+1}) \geq (f_1, v - \omega_{1h}^{n+1}) \quad \forall v \in V_{h_1}^{(u_2^n)}, \\ \omega_{1h}^{n+1} \leq r_h\psi, \qquad v \leq r_h\psi \end{gathered} \tag{3.1}$$

and (ω_{2h}^{n+1}) such that $\omega_{2h}^{n+1} \in V_{h_2}^{(u_1^{n+1})}$ solves

$$\begin{gathered} a_1(\omega_{2h}^{n+1}, v - \omega_{2h}^{n+1}) \geq (f_2, v - \omega_{2h}^{n+1}) \quad \forall v \in V_{h_2}^{(u_1^{n+1})}, \\ \omega_{2h}^{n+1} \leq r_h\psi, \qquad v \leq r_h\psi. \end{gathered} \tag{3.2}$$

Note that ω_{ih}^{n+1} is the finite element approximation of u_i^{n+1} defined in (2.31), (2.32).

Notation 1. From now on, we will adopt the following notations:

$$
\begin{gathered}
|\cdot|_1 = \|\cdot\|_{L^\infty(\Gamma_1)}, \qquad |\cdot|_2 = \|\cdot\|_{L^\infty(\Gamma_2)}, \\
\|\cdot\|_1 = \|\cdot\|_{L^\infty(\Omega_1)}, \qquad \|\cdot\|_2 = \|\cdot\|_{L^\infty(\Omega_2)}, \\
\pi_{h_1} = \pi_{h_2} = \pi_h.
\end{gathered}
\tag{3.3}
$$

The following lemma will play a key role in proving the main result of this paper.

Lemma 3.1.

$$
\begin{gathered}
\|u_1^{n+1} - u_{1h}^{n+1}\|_1 \le \sum_{p=1}^{n+1} \|u_1^p - \omega_{1h}^p\|_1 + \sum_{p=0}^{n} \|u_2^p - \omega_{2h}^p\|_2, \\
\|u_2^{n+1} - u_{2h}^{n+1}\|_2 \le \sum_{p=0}^{n+1} \|u_2^p - \omega_{2h}^p\|_2 + \sum_{p=1}^{n+1} \|u_1^p - \omega_{1h}^p\|_1.
\end{gathered}
\tag{3.4}
$$

Proof. The proof will be carried out by induction. In order to simplify the notations, we will take $h_1 = h_2 = h$.

Indeed, for $n = 1$, using the discrete version of Remark 2.3, we get

$$
\begin{aligned}
\|u_1^1 - u_{1h}^1\|_1 &\le \|u_1^1 - \omega_{1h}^1\|_1 + \|\omega_{1h}^1 - u_{1h}^1\|_1 \le \|u_1^1 - \omega_{1h}^1\|_1 + |\pi_h u_2^0 - \pi_h u_{2h}^0|_1 \\
&\le \|u_1^1 - \omega_{1h}^1\|_1 + |u_2^0 - u_{2h}^0|_1 \le \|u_1^1 - \omega_{1h}^1\|_1 + \|u_2^0 - u_{2h}^0\|_2, \\
\|u_2^1 - u_{2h}^1\|_2 &\le \|u_2^1 - \omega_{2h}^1\|_2 + \|\omega_{2h}^1 - u_{2h}^1\|_2 \le \|u_2^1 - \omega_{2h}^1\|_2 + |\pi_h u_1^1 - \pi_h u_{1h}^1|_2 \\
&\le \|u_2^1 - \omega_{2h}^1\|_2 + |u_1^1 - u_{1h}^1|_2 \le \|u_2^1 - \omega_{2h}^1\|_2 + \|u_1^1 - u_{1h}^1\|_1 \\
&\le \|u_2^1 - \omega_{2h}^1\|_2 + \|u_1^1 - \omega_{1h}^1\|_1 + \|u_2^0 - u_{2h}^0\|_2
\end{aligned}
\tag{3.5}
$$

so

$$
\begin{gathered}
\|u_1^1 - u_{1h}^1\|_1 \le \sum_{p=1}^{1} \|u_1^p - \omega_{1h}^p\|_1 + \sum_{p=0}^{0} \|u_2^p - \omega_{2h}^p\|_2, \\
\|u_2^1 - u_{2h}^1\|_2 \le \sum_{p=0}^{1} \|u_2^p - \omega_{2h}^p\|_2 + \sum_{p=1}^{1} \|u_1^p - \omega_{1h}^p\|_1.
\end{gathered}
\tag{3.6}
$$

For $n = 2$, using the discrete version of Remark 2.3, we have

$$
\begin{aligned}
\|u_1^2 - u_{1h}^2\|_1 &\le \|u_1^2 - \omega_{1h}^2\|_1 + \|\omega_{1h}^2 - u_{1h}^2\|_1 \le \|u_1^2 - \omega_{1h}^2\|_1 + |\pi_h u_2^1 - \pi_h u_{2h}^1|_1 \\
&\le \|u_1^2 - \omega_{1h}^2\|_1 + |u_2^1 - u_{2h}^1|_1 \le \|u_1^2 - \omega_{1h}^2\|_1 + \|u_2^1 - u_{2h}^1\|_2 \\
&\le \|u_1^2 - \omega_{1h}^2\|_1 + \|u_2^1 - \omega_{2h}^1\|_2 + \|u_1^1 - \omega_{1h}^1\|_1 + \|u_2^0 - u_{2h}^0\|_2, \\
\|u_2^2 - u_{2h}^2\|_2 &\le \|u_2^2 - \omega_{2h}^2\|_2 + \|\omega_{2h}^2 - u_{2h}^2\|_2 \le \|u_2^2 - \omega_{2h}^2\|_2 + |\pi_h u_1^2 - \pi_h u_{1h}^2|_1 \\
&\le \|u_2^2 - \omega_{2h}^2\|_2 + |u_1^2 - u_{1h}^2|_1 \le \|u_2^2 - \omega_{2h}^2\|_2 + \|u_1^2 - u_{1h}^2\|_2 \\
&\le \|u_2^2 - \omega_{2h}^2\|_2 + \|u_1^2 - \omega_{1h}^2\|_1 + \|u_2^1 - \omega_{2h}^1\|_2 + \|u_1^1 - \omega_{1h}^1\|_1 + \|u_2^0 - u_{2h}^0\|_2.
\end{aligned}
\tag{3.7}
$$

So

$$\begin{aligned}||u_1^2 - u_{1h}^2||_1 &\leq \sum_{p=1}^{2} ||u_1^p - \omega_{1h}^p||_1 + \sum_{p=0}^{1} ||u_2^p - \omega_{2h}^p||_2 \\ ||u_2^2 - u_{2h}^2||_1 &\leq \sum_{p=0}^{2} ||u_2^p - \omega_{2h}^p||_2 + \sum_{p=1}^{2} ||u_1^p - \omega_{1h}^p||_1.\end{aligned} \tag{3.8}$$

Let us now suppose that

$$||u_2^n - u_{2h}^n||_2 \leq \sum_{p=0}^{n} ||u_2^p - \omega_{2h}^p||_2 + \sum_{p=1}^{n} ||u_1^p - \omega_{1h}^p||_1. \tag{3.9}$$

Then, using the discrete version of Remark 2.3 again, we get

$$\begin{aligned}||u_1^{n+1} - u_{1h}^{n+1}||_1 &\leq ||u_1^{n+1} - \omega_{1h}^{n+1}||_1 + ||\omega_{1h}^{n+1} - u_{1h}^{n+1}||_1 \leq ||u_1^{n+1} - \omega_{1h}^{n+1}||_1 + |\pi_h u_2^n - \pi_h u_{2h}^n|_1 \\ &\leq ||u_1^{n+1} - \omega_{1h}^{n+1}||_1 + |u_2^n - u_{2h}^n|_1 \leq ||u_1^{n+1} - \omega_{1h}^{n+1}||_1 + ||u_2^n - u_{2h}^n||_2 \\ &\leq ||u_1^{n+1} - \omega_{1h}^{n+1}||_1 + \sum_{p=0}^{n} ||u_2^p - \omega_{2h}^p||_2 + \sum_{p=1}^{n} ||u_1^p - \omega_{1h}^p||_1\end{aligned} \tag{3.10}$$

and consequently,

$$||u_1^{n+1} - u_{1h}^{n+1}||_1 \leq \sum_{p=1}^{n+1} ||u_1^p - \omega_{1h}^p||_1 + \sum_{p=0}^{n} ||u_2^p - \omega_{2h}^p||_2. \tag{3.11}$$

Likewise, using the above estimate, we get

$$\begin{aligned}||u_2^{n+1} - u_{2h}^{n+1}||_2 &\leq ||u_2^{n+1} - \omega_{2h}^{n+1}||_2 + ||\omega_{2h}^{n+1} - u_{2h}^{n+1}||_2 \leq ||u_2^{n+1} - \omega_{2h}^{n+1}||_2 \\ &\quad + |\pi_h u_1^{n+1} - \pi_h u_{1h}^{n+1}|_2 \leq ||u_2^{n+1} - \omega_{2h}^{n+1}||_2 + |u_1^{n+1} - u_{1h}^{n+1}|_2 \\ &\leq ||u_2^{n+1} - \omega_{2h}^{n+1}||_2 + ||u_1^{n+1} - u_{1h}^{n+1}||_1 \leq ||u_2^{n+1} - \omega_{2h}^{n+1}||_2 \\ &\quad + \sum_{p=1}^{n+1} ||u_1^p - \omega_{1h}^p||_1 + \sum_{p=0}^{n} ||u_2^p - \omega_{2h}^p||_2.\end{aligned} \tag{3.12}$$

Hence,

$$||u_2^{n+1} - u_{2h}^{n+1}||_2 \leq \sum_{p=0}^{n+1} ||u_2^p - \omega_{2h}^p||_2 + \sum_{p=1}^{n+1} ||u_1^p - \omega_{1h}^p||_1. \tag{3.13}$$

□

3.2. L^∞-error estimate.

Theorem 3.2. *Let* $h = \max(h_1, h_2)$. *Then, there exists a constant* C *independent of both* h *and* n *such that*

$$||u_i - u_{ih}^{n+1}||_{L^\infty(\Omega_i)} \leq Ch^2 |\log h|^3; \quad i = 1, 2. \tag{3.14}$$

Proof. Let us give the proof for $i = 1$. The case $i = 2$ is similar.

Indeed, let $\kappa = \max(k_1, k_2)$, then

$$\begin{aligned} ||u_1 - u_{1h}^{n+1}||_1 &\le ||u_1 - u_1^{n+1}||_1 + ||u_1^{n+1} - u_{1h}^{n+1}||_1 \\ &\le \kappa^{2n} |u^0 - u|_1 + ||u_1^{n+1} - u_{1h}^{n+1}||_1 \\ &\le \kappa^{2n} |u^0 - u|_1 + \sum_{p=1}^{n+1} ||u_1^p - \omega_{1h}^p||_1 + \sum_{p=0}^{n} ||u_2^p - \omega_{2h}^p||_2 \\ &\le \kappa^{2n} |u^0 - u|_1 + 2(n+1)Ch^2 |\log h|^2, \end{aligned} \tag{3.15}$$

where we have used Theorem 2.5, Lemma 3.1, and Theorem 2.4, respectively.

Now setting

$$\kappa^{2n} \le h^2, \tag{3.16}$$

we obtain

$$||u_1 - u_{1h}^{n+1}||_1 \le Ch^2 |\log h|^3, \tag{3.17}$$

which is the desired error estimate. □

3.3. The equation case. The analysis developed above remains valid for the equation problem ($\psi = \infty$). Consequently, the error estimate (3.14) becomes

$$||u_i - u_{ih}^{n+1}||_{L^\infty(\Omega_i)} \le Ch^2 |\log h|^2; \quad i = 1, 2. \tag{3.18}$$

Remark 3.3. The reduction constant k can be quite close to one if the overlapping region is thin. Therefore, to ensure a good accuracy of the approximation, this region must be large enough.

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M. Boulbrachene: Department of Mathematics, College of Science, Sultan Qaboos University, P.O. Box 36, Muscat 123, Oman
E-mail address: boulbrac@squ.edu.om

S. Saadi: Departement de Mathematiques, Faculte des Sciences, Universite Badji Mokhtar, BP 12 Annaba, Algerie
E-mail address: signor_2000@yahoo.fr

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José Roberto Castilho Piqueira, Telecommunication and Control Engineering Department, Polytechnic School, The University of São Paulo, 05508-970 São Paulo, Brazil; piqueira@lac.usp.br

Elbert E. Neher Macau, Laboratório Associado de Matemática Aplicada e Computação (LAC), Instituto Nacional de Pesquisas Espaciais (INPE), São Josè dos Campos, 12227-010 São Paulo, Brazil ; elbert@lac.inpe.br

Celso Grebogi, Center for Applied Dynamics Research, King's College, University of Aberdeen, Aberdeen AB24 3UE, UK; grebogi@abdn.ac.uk