Research Article
On Bloch-Type Functions with Hadamard Gaps

Stevo Stević

We give some sufficient and necessary conditions for an analytic function f on the unit ball B with Hadamard gaps, that is, for $f(z) = \sum_{k=1}^{\infty} P_{n_k}(z)$ (the homogeneous polynomial expansion of f) satisfying $n_{k+1}/n_k \geq \lambda > 1$ for all $k \in \mathbb{N}$, to belong to the space $\mathcal{B}_p^{\alpha}(B) = \{f \mid \sup_{0<r<1}(1-r^2)^{\alpha} \|\mathcal{R}f_r\|_p < \infty, f \in H(B)\}$, $p = 1, 2, \infty$ as well as to the corresponding little space. A remark on analytic functions with Hadamard gaps on mixed norm space on the unit disk is also given.

1. Introduction

Let $B = \{z \in \mathbb{C}^n : |z| < 1\}$ be the open unit ball of $\mathbb{C}^n$, $\partial B = \{z \in \mathbb{C}^n : |z| = 1\}$ its boundary, $\mathbb{D}$ the unit disk in $\mathbb{C}$, dv the normalized Lebesgue measure of B (i.e., $v(B) = 1$), and $d\sigma$ the normalized rotation invariant Lebesgue measure of S satisfying $\sigma(\partial B) = 1$. We denote the class of all holomorphic functions on the unit ball by $H(B)$.

For $f \in H(B)$ with the Taylor expansion $f(z) = \sum_{|\beta| \geq 0} a_\beta z^\beta$, let $\mathcal{R}f(z) = \sum_{|\beta| \geq 0} |\beta| a_\beta z^\beta$ be the radial derivative of f, where $\beta = (\beta_1, \beta_2, \ldots, \beta_n)$ is a multi-index and $z^\beta = z_1^{\beta_1} \cdots z_n^{\beta_n}$. It is well known that $\mathcal{R}f(z) = \sum_{j=1}^{n} z_j (\partial f/\partial z_j)(z) = \sum_{k=0}^{\infty} k P_k(z)$, if $f(z) = \sum_{k=0}^{\infty} P_k(z)$.

As usual, we write

$$\|f_r\|_p = \left(\int_S |f(r\zeta)|^p d\sigma(\zeta) \right)^{1/p} \tag{1.1}$$

if $p \in (0, \infty)$, and where $f_r(\zeta) = f(r\zeta)$. If $p = \infty$, then $\|f\|_\infty = \sup_{z \in B} |f(z)|$.

Let $\alpha > 0$. The α-Bloch space $\mathscr{B}^\alpha = \mathscr{B}^\alpha(B)$ is the space of all holomorphic functions f on B such that

$$b_\alpha(f) = \sup_{z \in B} \left(1 - |z|^2\right)^\alpha |\mathscr{R}f(z)| < \infty. \tag{1.2}$$

It is clear that $\mathscr{B}^\alpha$ is a normed space under the norm $\|f\|_{\mathscr{B}^\alpha} = |f(0)| + b_\alpha(f)$, and $\mathscr{B}^{\alpha_1} \subset \mathscr{B}^{\alpha_2}$ for $\alpha_1 < \alpha_2$. Let $\mathscr{B}_0^\alpha$ denote the subspace of $\mathscr{B}^\alpha$ consisting of those $f \in \mathscr{B}^\alpha$ for which $(1 - |z|^2)^\alpha |\mathscr{R}f(z)| \to 0$ as $|z| \to 1$. This space is called the little α-Bloch space. For $\alpha = 1$, the α-Bloch space and the little α-Bloch space become Bloch space $\mathscr{B}$ and the little Bloch space $\mathscr{B}_0$. Some characterizations of these spaces can be found, for example, in the following papers [1–6].

We say that an analytic function f on the unit disk $\mathbb{D}$ has Hadamard gaps if $f(z) = \sum_{k=1}^\infty a_k z^{n_k}$ where $n_{k+1}/n_k \geq \lambda > 1$, for all $k \in \mathbb{N}$.

In [7], Yamashita proved the following result.

THEOREM 1.1. *Assume that f is an analytic function on $\mathbb{D}$ with Hadamard gaps. Then for $\alpha > 0$, the following two propositions hold:*
   (a) $f \in \mathscr{B}^\alpha(\mathbb{D})$ *if and only if* $\limsup_{k \to \infty} |a_k| n_k^{1-\alpha} < \infty$;
   (b) $f \in \mathscr{B}_0^\alpha(\mathbb{D})$ *if and only if* $\lim_{k \to \infty} |a_k| n_k^{1-\alpha} = 0$.

An analytic function on B with the homogeneous expansion $f(z) = \sum_{k=1}^\infty P_{n_k}(z)$ (here, P_{n_k} is a homogeneous polynomial of degree n_k) is said to have Hadamard gaps if $n_{k+1}/n_k \geq \lambda > 1$, for all $k \in \mathbb{N}$. In [8], among others, Choa generalizes the main result in [9], proving the following result.

THEOREM 1.2. *Assume that $p \in (0, \infty)$ and $f(z) = \sum_{k=1}^\infty P_{n_k}(z)$ is an analytic function on B with Hadamard gaps. Then the following statements are equivalent:*
   (a) $\|f\|_{X_p} = (\int_B |\mathscr{R}f(z)|^p (1 - |z|^2)^{p-1} dv(z))^{1/p} < \infty$;
   (b) $\sum_{k=1}^\infty \|P_{n_k}\|_p^p < \infty$.

This result motivates us to find some characterizations for certain function spaces of analytic functions on the unit ball, in terms of the sequence $(\|P_{n_k}\|_p)_{k \in \mathbb{N}}$.

Now note that the quantity b_α in the definition of the α-Bloch spaces can be written in the following form:

$$b_\alpha(f) = \sup_{0 < r < 1} \left(1 - r^2\right)^\alpha \sup_{\zeta \in S} |\mathscr{R}f(r\zeta)| = \sup_{0 < r < 1} \left(1 - r^2\right)^\alpha M_\infty(\mathscr{R}f, r). \tag{1.3}$$

On the other hand, the quantity b_α can be considered as the limit case of the following quantities:

$$\|f\|_{\mathscr{B}_p^\alpha} = \sup_{0 < r < 1} \left(1 - r^2\right)^\alpha \|\mathscr{R}f_r\|_p, \tag{1.4}$$

as $p \to \infty$. Note that for every $f \in H(B)$ and $p \in (0, \infty)$,

$$\sup_{0 < r < 1} \left(1 - r^2\right)^\alpha \|\mathscr{R}f_r\|_p \leq \sup_{0 < r < 1} \left(1 - r^2\right)^\alpha \|\mathscr{R}f_r\|_\infty. \tag{1.5}$$

Hence, in this paper we also consider analytic functions with Hadamard gaps on the following spaces:

$$
\begin{aligned}
\mathcal{B}_p^\alpha &= \left\{ f \mid \sup_{0<r<1} (1-r^2)^\alpha \|\mathcal{R}f_r\|_p < \infty,\ f \in H(B) \right\}, \\
\mathcal{B}_{p,0}^\alpha &= \left\{ f \mid \lim_{r\to1}(1-r^2)^\alpha \|\mathcal{R}f_r\|_p = 0,\ f \in H(B) \right\}.
\end{aligned}
\tag{1.6}
$$

Motivated by Theorem 1.1 in this paper, we study analytic functions with Hadamard gaps, which belong to $\mathcal{B}_p^\alpha$ or $\mathcal{B}_{p,0}^\alpha$ space when $p = 1, 2, \infty$. Some characterizations for these classes of functions on the unit ball are given in terms of the sequence $(\|P_{n_k}\|_p)_{k\in\mathbb{N}}$. The following are the main results.

THEOREM 1.3. *Assume that $\alpha > 0$, $p = 1, 2, \infty$, and $f(z) = \sum_{k=1}^\infty P_{n_k}(z)$ is an analytic function on B with Hadamard gaps. Then the following statements are equivalent:*

(a) $f \in \mathcal{B}_p^\alpha$;

(b) $\limsup_{k\to\infty} \|P_{n_k}\|_p n_k^{1-\alpha} < \infty$.

THEOREM 1.4. *Assume that $\alpha > 0$, $p = 1, 2, \infty$, and $f(z) = \sum_{k=1}^\infty P_{n_k}(z)$ is an analytic function on B with Hadamard gaps. Then the following statements are equivalent:*

(a) $f \in \mathcal{B}_{p,0}^\alpha$;

(b) $\lim_{k\to\infty} \|P_{n_k}\|_p n_k^{1-\alpha} = 0$.

Throughout this paper, constants are denoted by C, they are positive and may differ from one occurrence to the other. The notation $A \asymp B$ means that there is a positive constant C such that $B/C \leq A \leq CB$.

2. Proof of main results

Before proving the main results of this paper we quote two auxiliary results which are incorporated in the lemmas which follow (see [9, 10]).

LEMMA 2.1. *Assume that $p \in (0, \infty)$. If (n_k) is an increasing sequence of positive integers satisfying $n_{k+1}/n_k \geq \lambda > 1$, for all k, then there is a positive constant A depending only on p and λ such that*

$$
\frac{1}{A}\left(\sum_{k=1}^\infty |a_k|^2 \right)^{1/2} \leq \left(\frac{1}{2\pi} \int_0^{2\pi} \left| \sum_{k=1}^\infty a_k e^{in_k\theta} \right|^p d\theta \right)^{1/p} \leq A \left(\sum_{k=1}^\infty |a_k|^2 \right)^{1/2}
\tag{2.1}
$$

for any number a_k, $k \in \mathbb{N}$.

LEMMA 2.2. *Assume that $\alpha > 0$, $p > 0$, $n \in \mathbb{N}_0$, $(a_n)_{n\in\mathbb{N}_0}$ is the sequence of nonnegative numbers, $I_n = \{k \mid 2^n \leq k < 2^{n+1},\ k \in \mathbb{N}\}$, $t_n = \sum_{k\in I_n} a_k$, and $g(x) = \sum_{n=1}^\infty a_n x^n$. Then there is a positive constant K depending only on p and α such that*

$$
\frac{1}{K} \sum_{n=0}^\infty \frac{t_n^p}{2^{n\alpha}} \leq \int_0^1 (1-x)^{\alpha-1} g^p(x)\,dx \leq K \sum_{n=0}^\infty \frac{t_n^p}{2^{n\alpha}}.
\tag{2.2}
$$

Proof of Theorem 1.3. (a)$\Rightarrow$(b) (Case $p = 1$). Let $f \in \mathcal{B}_1^\alpha$. Let $f_\zeta(w) = f(\zeta w)$, $\zeta \in S$, where ζ is fixed and $w \in \mathbb{D}$, be a slice function. By some calculation we see that

$$f_\zeta'(w) = \zeta_1 \frac{\partial f}{\partial z_1}(w\zeta) + \cdots + \zeta_n \frac{\partial f}{\partial z_n}(w\zeta) = \frac{1}{w}\mathcal{R}f(w\zeta). \tag{2.3}$$

From (2.3) and since $f_\zeta'(w) = \sum_{k=1}^\infty n_k P_{n_k}(\zeta) w^{n_k-1}$, we have that

$$
\begin{aligned}
\int_S n_k |P_{n_k}(\zeta)| \, d\sigma(\zeta) &= \int_S \left| \frac{1}{2\pi i} \int_{\partial r\mathbb{D}} \frac{\eta f_\zeta'(\eta)}{\eta^{n_k+1}} d\eta \right| d\sigma(\zeta) \\
&\le \frac{1}{2\pi} \int_{\partial r\mathbb{D}} \int_S \frac{|\mathcal{R}f(\zeta\eta)|}{|\eta^{n_k+1}|} d\sigma(\zeta) |d\eta| \\
&\le \frac{\|f_r\|_{\mathcal{B}_1^\alpha}}{(1-r)^\alpha r^{n_k}},
\end{aligned}
\tag{2.4}
$$

which implies that

$$n_k r^{n_k} \|P_{n_k}\|_1 \le \frac{\|f\|_{\mathcal{B}_1^\alpha}}{(1-r)^\alpha}, \tag{2.5}$$

for every $k \in \mathbb{N}$ and $r \in (0,1)$. Choosing $r = 1 - (1/n_k)$, we obtain $n_k^{1-\alpha}\|P_{n_k}\|_1 \le C$, as desired.

(b)$\Rightarrow$(a) (Case $p = 1$). Assume $\limsup_{k\to\infty} \|P_{n_k}\|_1 n_k^{1-\alpha} < \infty$. We have that

$$
\begin{aligned}
\|f\|_{\mathcal{B}_1^\alpha} &= \sup_{0<r<1} (1-r^2)^\alpha \int_S |\mathcal{R}f(r\zeta)| \, d\sigma(\zeta) \\
&= \sup_{0<r<1} (1-r^2)^\alpha \int_S \left| \sum_{k=1}^\infty n_k P_{n_k}(\zeta) r^{n_k} \right| d\sigma(\zeta) \\
&\le \sup_{0<r<1} (1-r^2)^\alpha \sum_{k=1}^\infty n_k \|P_{n_k}\|_1 r^{n_k} \\
&\le \sup_{0<r<1} (1-r^2)^{\alpha+1} \sum_{n=1}^\infty \left(\sum_{n_k \le n} n_k \|P_{n_k}\|_1 \right) r^n \\
&\le C \sup_{0<r<1} (1-r^2)^{\alpha+1} \sum_{n=1}^\infty \left(\sum_{n_k \le n} n_k^\alpha \right) r^n \\
&\le C \sup_{0<r<1} (1-r^2)^{\alpha+1} \sum_{n=1}^\infty n^\alpha r^n \le C,
\end{aligned}
\tag{2.6}
$$

where we have used the fact that there is a positive constant C independent of n such that $\sum_{n_k \le n} n_k^\alpha \le Cn^\alpha$ (here is used the assumption that $n_{k+1}/n_k \ge \lambda > 1$) and the following well-known estimate:

$$\sum_{n=1}^\infty n^\alpha r^n \le C(1-r)^{-(\alpha+1)}, \tag{2.7}$$

$\alpha > 0$, $r \in [0,1)$; see, for example, [11].

Case $p = 2$. Since

$$\|f\|_{\mathscr{B}_2^\alpha} = \sup_{0<r<1} (1-r^2)^\alpha \left(\sum_{k=1}^\infty n_k^2 \|P_{n_k}\|_2^2 r^{2n_k} \right)^{1/2} \tag{2.8}$$

we have that

$$\sup_{0<r<1} (1-r^2)^\alpha n_k \|P_{n_k}\|_2 r^{n_k} \le \|f\|_{\mathscr{B}_2^\alpha} \le \sup_{0<r<1} (1-r^2)^\alpha \sum_{k=1}^\infty n_k \|P_{n_k}\|_2 r^{n_k}, \tag{2.9}$$

from which the result follows similar to the case $p = 1$.

Now we show that $(a) \Leftrightarrow (b)$ for case $p = \infty$. As above, the function $f_\zeta(w) = \sum_{k=1}^\infty P_{n_k}(\zeta) w^{n_k}$, where $w = re^{i\theta}$, is a lacunary series in $\mathbb{D}$ and

$$(1-r^2)^\alpha \mathscr{R} f(r\zeta) = re^{i\theta} (1-r^2)^\alpha f'_{\zeta e^{-i\theta}}(re^{i\theta}), \tag{2.10}$$

from which by Theorem 1.1 the equivalence follows. □

Proof of Theorem 1.4. $(a) \Rightarrow (b)$ (Case $p = 1$). Let $f \in \mathscr{B}_{1,0}^\alpha$, then for every $\varepsilon > 0$ there is a $\delta > 0$ such that

$$(1-r^2)^\alpha \int_S |\mathscr{R} f(r\zeta)| \, d\sigma(\zeta) < \varepsilon, \tag{2.11}$$

whenever $\delta < r < 1$. From (2.4), (2.11), and rotational invariance of $d\sigma$, we have that

$$\begin{aligned}
\int_S n_k |P_{n_k}(\zeta)| \, d\sigma(\zeta) &\le \frac{1}{2\pi} \int_{\partial r\mathbb{D}} \int_S \frac{|\mathscr{R} f(\zeta\eta)|}{|\eta^{n_k+1}|} \, d\sigma(\zeta)|d\eta| \\
&\le \frac{1}{2\pi} \int_{\partial r\mathbb{D}} \int_S \frac{(1-r^2)^\alpha |\mathscr{R} f(\zeta\eta)|}{(1-r^2)^\alpha r^{n_k+1}} \, d\sigma(\zeta)|d\eta| \\
&\le \frac{\varepsilon}{(1-r)^\alpha r^{n_k}},
\end{aligned} \tag{2.12}$$

which implies that

$$n_k r^{n_k} \|P_{n_k}\|_1 \le \frac{\varepsilon}{(1-r)^\alpha} \tag{2.13}$$

for every $k \in \mathbb{N}$ and $r \in (\delta, 1)$. Choosing $r = 1 - (1/n_k)$, we obtain

$$n_k \|P_{n_k}\|_1 \le C\varepsilon n_k^\alpha, \tag{2.14}$$

from which (b) follows in this case.

$(b) \Rightarrow (a)$ (Case $p = 1$). Assume that $\lim_{k \to \infty} \|P_{n_k}\|_1 n_k^{1-\alpha} = 0$, then for every $\varepsilon > 0$ there is a $k_0 \in \mathbb{N}$ such that

$$\|P_{n_k}\|_1 \le \varepsilon n_k^{\alpha-1}, \quad \text{for } k \ge k_0. \tag{2.15}$$

We may assume that $k_0 = 1$. From this and by the proof of Theorem 1.3, (b)$\Rightarrow$(a) (Case $p = 1$), we have that

$$
\begin{aligned}
(1-r^2)^\alpha \|\mathscr{R}f_r\|_1 &\leq \sup_{0<r<1} (1-r^2)^{\alpha+1} \sum_{n=1}^\infty \left(\sum_{n_k \leq n} n_k \|P_{n_k}\|_1 \right) r^n \\
&\leq C\varepsilon \sup_{0<r<1} (1-r^2)^{\alpha+1} \sum_{n=1}^\infty \left(\sum_{n_k \leq n} n_k^\alpha \right) r^n \\
&\leq C\varepsilon \sup_{0<r<1} (1-r^2)^{\alpha+1} \sum_{n=1}^\infty n^\alpha r^n \leq C\varepsilon,
\end{aligned}
\tag{2.16}
$$

from which the implication follows.

Case $p = 2$. By using (2.9) the result follows similar to the Case $p = 1$. The proof is omitted.

Finally, in view of (2.10) and employing Theorem 1.1(b) it is easy to see that $(a)\Leftrightarrow(b)$ for case $p = \infty$. $\qquad\square$

3. The case of mixed norm space

In this section, we give a note concerning analytic functions with Hadamard gaps on the mixed norm space. The mixed norm space $H_{p,q,\alpha}(B)$, $p,q > 0$, and $\alpha \in (-1, \infty)$, consists of all $f \in H(B)$ such that

$$
\|f\|_{p,q,\alpha} = \left(\int_0^1 \|f(r\zeta)\|_p^q (1-r)^\alpha dr \right)^{1/q} < \infty.
\tag{3.1}
$$

From [12, Theorem 4] the following result holds.

THEOREM 3.1. *Assume that $p \in (0, \infty)$, $\alpha > -1$ and $f(z) = \sum_{k=1}^\infty a_k z^{n_k}$ is an analytic function on $\mathbb{D}$ with Hadamard gaps. Then $f^{(m)} \in H_{p,q,\alpha}(\mathbb{D})$ if and only if $\sum_{k=0}^\infty n_k^{qm-\alpha-1} |a_k|^q < \infty$.*

Proof. First we consider the case $m = 0$. Similar to the proof of [12, Theorem 4] and by Lemmas 2.1 and 2.2, we have that

$$
\begin{aligned}
\|f\|_{H_{p,q,\alpha}}^q &= \int_0^1 \left(\frac{1}{2\pi} \int_0^{2\pi} \left| \sum_{k=1}^\infty a_k r^{n_k} e^{in_k\theta} \right|^p d\theta \right)^{q/p} (1-r)^\alpha dr \\
&\asymp \int_0^1 \left(\sum_{k=1}^\infty |a_k|^2 r^{2n_k} \right)^{q/2} (1-r)^\alpha dr \\
&\asymp \int_0^1 \left(\sum_{k=1}^\infty |a_k|^2 \rho^{n_k} \right)^{q/2} (1-\rho)^\alpha d\rho \\
&\asymp \sum_{k=0}^\infty \frac{1}{2^{(\alpha+1)k}} \left(\sum_{m \in I_k} |a_m|^2 \right)^{q/2} \asymp \sum_{k=0}^\infty \frac{|a_k|^q}{n_k^{\alpha+1}},
\end{aligned}
\tag{3.2}
$$

from which the result follows in this case.

Since f has Hadamard gaps and $f^{(m)}(z) = \sum_{k=1}^{\infty} a_k n_k (n_k - 1) \cdots (n_k - m + 1) z^{n_k - m}$, it follows that $f^{(m)}$ has Hadamard gaps too. Applying the just proved result to the function $f^{(m)}$, we obtain that $f^{(m)} \in H_{p,q,\alpha}(\mathbb{D})$ if and only if

$$\sum_{k=0}^{\infty} \frac{\left| n_k (n_k - 1) \cdots (n_k - m + 1) a_k \right|^q}{n_k^{\alpha+1}} \asymp \sum_{k=0}^{\infty} \frac{\left| a_k \right|^q}{n_k^{\alpha+1-mq}} < \infty, \tag{3.3}$$

finishing the proof. $\qquad\square$

Remark 3.2. Motivated by [12, Theorems 3 and 4], we can conjecture that if $p \in (0, \infty)$, $\alpha > -1$, and $f(z) = \sum_{k=1}^{\infty} P_{n_k}(z)$ is an analytic function on B with Hadamard gaps, then $\mathscr{R}^{(m)} f \in H_{p,q,\alpha}(B)$ if and only if $\sum_{k=0}^{\infty} n_k^{qm-\alpha-1} \|P_{n_k}\|_p^q < \infty$. Note that the result is true for the case of the weighted Bergman space, that is, when $p = q$, see [12, Corollary 1]. It is also expected that Theorems 1.3 and 1.4 hold for every $p \in [1; \infty]$ (for the case $n = 1$, see [13]).

Acknowledgment

References

[1] D. D. Clahane and S. Stević, "Norm equivalence and composition operators between Bloch/Lipschitz spaces of the ball," *Journal of Inequalities and Applications*, vol. 2006, Article ID 61018, 11 pages, 2006.

[2] S. Li, "Fractional derivatives of Bloch-type functions," *Siberian Mathematical Journal*, vol. 46, no. 2, pp. 394–402, 2005.

[3] M. Nowak, "Bloch space on the unit ball of $\mathbb{C}^n$," *Annales Academiæ Scientiarium Fennicæ*, vol. 23, no. 2, pp. 461–473, 1998.

[4] M. Nowak, "Bloch space and Möbius invariant Besov spaces on the unit ball of $\mathbb{C}^n$," *Complex Variables, Theory and Application*, vol. 44, no. 1, pp. 1–12, 2001.

[5] C. H. Ouyang, W. S. Yang, and R. H. Zhao, "Characterizations of Bergman spaces and Bloch space in the unit ball of $\mathbb{C}^n$," *Transactions of the American Mathematical Society*, vol. 347, no. 11, pp. 4301–4313, 1995.

[6] R. M. Timoney, "Bloch functions in several complex variables. I," *The Bulletin of the London Mathematical Society*, vol. 12, no. 4, pp. 241–267, 1980.

[7] S. Yamashita, "Gap series and α-Bloch functions," *Yokohama Mathematical Journal*, vol. 28, no. 1-2, pp. 31–36, 1980.

[8] J. S. Choa, "Some properties of analytic functions on the unit ball with Hadamard gaps," *Complex Variables, Theory and Application*, vol. 29, no. 3, pp. 277–285, 1996.

[9] J. Miao, "A property of analytic functions with Hadamard gaps," *Bulletin of the Australian Mathematical Society*, vol. 45, no. 1, pp. 105–112, 1992.

[10] M. Mateljević and M. Pavlović, "L^p-behavior of power series with positive coefficients and Hardy spaces," *Proceedings of the American Mathematical Society*, vol. 87, no. 2, pp. 309–316, 1983.

[11] A. Zygmund, *Trigonometric Series. Vol. I*, Cambridge University Press, New York, NY, USA, 2nd edition, 1959.

[12] S. Stević, "A generalization of a result of Choa on analytic functions with Hadamard gaps," *Journal of the Korean Mathematical Society*, vol. 43, no. 3, pp. 579–591, 2006.

[13] D. Girela and J. A. Peláez, "Integral means of analytic functions," *Ann. Acad. Sci. Fenn.*, vol. 29, pp. 459–469, 2004.

Stevo Stević: Mathematical Institute of the Serbian Academy of Science, Knez Mihailova 35/I, 11000 Beograd, Serbia
Email addresses: sstevic@ptt.yu ;   sstevo@matf.bg.ac.yu

Special Issue on
Decision Support for Intermodal Transport

Call for Papers

Intermodal transport refers to the movement of goods in a single loading unit which uses successive various modes of transport (road, rail, water) without handling the goods during mode transfers. Intermodal transport has become an important policy issue, mainly because it is considered to be one of the means to lower the congestion caused by single-mode road transport and to be more environmentally friendly than the single-mode road transport. Both considerations have been followed by an increase in attention toward intermodal freight transportation research.

Various intermodal freight transport decision problems are in demand of mathematical models of supporting them. As the intermodal transport system is more complex than a single-mode system, this fact offers interesting and challenging opportunities to modelers in applied mathematics. This special issue aims to fill in some gaps in the research agenda of decision-making in intermodal transport.

The mathematical models may be of the optimization type or of the evaluation type to gain an insight in intermodal operations. The mathematical models aim to support decisions on the strategic, tactical, and operational levels. The decision-makers belong to the various players in the intermodal transport world, namely, drayage operators, terminal operators, network operators, or intermodal operators.

Topics of relevance to this type of decision-making both in time horizon as in terms of operators are:

- Intermodal terminal design
- Infrastructure network configuration
- Location of terminals
- Cooperation between drayage companies
- Allocation of shippers/receivers to a terminal
- Pricing strategies
- Capacity levels of equipment and labour
- Operational routines and lay-out structure
- Redistribution of load units, railcars, barges, and so forth
- Scheduling of trips or jobs
- Allocation of capacity to jobs
- Loading orders
- Selection of routing and service

Before submission authors should carefully read over the journal's Author Guidelines, which are located at http://www.hindawi.com/journals/jamds/guidelines.html. Prospective authors should submit an electronic copy of their complete manuscript through the journal Manuscript Tracking System at http://mts.hindawi.com/, according to the following timetable:

Manuscript Due	June 1, 2009
First Round of Reviews	September 1, 2009
Publication Date	December 1, 2009

Lead Guest Editor

Gerrit K. Janssens, Transportation Research Institute (IMOB), Hasselt University, Agoralaan, Building D, 3590 Diepenbeek (Hasselt), Belgium; Gerrit.Janssens@uhasselt.be

Guest Editor

Cathy Macharis, Department of Mathematics, Operational Research, Statistics and Information for Systems (MOSI), Transport and Logistics Research Group, Management School, Vrije Universiteit Brussel, Pleinlaan 2, 1050 Brussel, Belgium; Cathy.Macharis@vub.ac.be