

ON THE EXISTENCE OF SOLUTIONS TO A FOURTH-ORDER QUASILINEAR RESONANT PROBLEM

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By means of Morse theory we prove the existence of a nontrivial solution to a superlinear p -harmonic elliptic problem with Navier boundary conditions having a linking structure around the origin. Moreover, in case of both resonance near zero and nonresonance at $+\infty$ the existence of two nontrivial solutions is shown.

1. Introduction and main results

Let $p > 1$ and $\Omega \subset \mathbb{R}^n$ be a smooth bounded domain with $n \geq 2p + 1$. We are concerned with the existence of nontrivial solutions to the p -harmonic equation

$$\Delta(|\Delta u|^{p-2}\Delta u) = g(x, u) \quad \text{in } \Omega \quad (1.1)$$

with Navier boundary conditions

$$u = \Delta u = 0 \quad \text{on } \partial\Omega, \quad (1.2)$$

where $g : \Omega \times \mathbb{R} \rightarrow \mathbb{R}$ is a Carathéodory function such that for some $C > 0$,

$$|g(x, s)| \leq C(1 + |s|^{q-1}) \quad (1.3)$$

for a.e. $x \in \Omega$ and all $s \in \mathbb{R}$, being $1 \leq q < p_*$ and $p_* = np/(n-2p)$.

It is well known that the functional $\Phi : W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega) \rightarrow \mathbb{R}$

$$\Phi(u) = \frac{1}{p} \int_{\Omega} |\Delta u|^p dx - \int_{\Omega} G(x, u) dx, \quad (1.4)$$

with $G(x, s) = \int_0^s g(x, t) dt$, is of class C^1 and

$$\langle \Phi'(u), \varphi \rangle = \int_{\Omega} |\Delta u|^{p-2} \Delta u \Delta \varphi dx - \int_{\Omega} g(x, u) \varphi dx \quad (1.5)$$

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for each $\varphi \in W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega)$. Moreover, the critical points of Φ are weak solutions for (1.1). Notice that for the eigenvalue problem

$$\Delta(|\Delta u|^{p-2}\Delta u) = \lambda|u|^{p-2}u \quad \text{in } \Omega \quad (1.6)$$

with boundary data (1.2), as for the p -Laplacian eigenvalue problem with Dirichlet boundary data,

$$\lambda_n = \inf_{A \in \Gamma_n} \sup_{u \in A} \int_{\Omega} |\Delta u|^p dx, \quad n = 1, 2, \dots \quad (1.7)$$

is the sequence of eigenvalues, where

$$\Gamma_n = \{A \subseteq W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega) \setminus \{0\} : A = -A, \gamma(A) \geq n\}, \quad (1.8)$$

being $\gamma(A)$ the Krasnoselski's genus of the set A . This follows by the Ljusternik-Schnirelman theory for C^1 -manifolds proved in [13] applied to the functional

$$\begin{aligned} J|_{\mathcal{M}}(u) &= \int_{\Omega} |\Delta u|^p dx, \\ \mathcal{M} &= \left\{ u \in W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega) : \int_{\Omega} |u|^p dx = 1 \right\}, \end{aligned} \quad (1.9)$$

since \mathcal{M} is a C^1 -manifold with tangent space

$$T_u \mathcal{M} = \left\{ w \in W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega) : \int_{\Omega} |u|^{p-2} uw dx = 0 \right\}. \quad (1.10)$$

The next remark is the starting point of our paper.

Remark 1.1. It has been recently proved by Drábek and Ôtani [4] that (1.6) with boundary data (1.2) has the least eigenvalue

$$\lambda_1(p) = \inf \left\{ \int_{\Omega} |\Delta u|^p dx : u \in W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega), \|u\|_p^p = 1 \right\} \quad (1.11)$$

which is simple, positive, and isolated in the sense that the solutions of (1.6) with $\lambda = \lambda_1(p)$ form a one-dimensional linear space spanned by a positive eigenfunction $\phi_1(p)$ associated with $\lambda_1(p)$ and there exists $\delta > 0$ so that $(\lambda_1(p), \lambda_1(p) + \delta)$ does not contain other eigenvalues. The situation is actually more involved with Dirichlet boundary conditions

$$u = \nabla u = 0 \quad \text{on } \partial\Omega \quad (1.12)$$

and, to our knowledge, it is not clear whether the first eigenspace has the previous good properties; the fact is that while Navier boundary conditions allow to reduce the fourth-order problem into a system of two second-order problems, Dirichlet boundary conditions do not. Some pathologies are indeed known, for instance, the first eigenfunction of $\Delta^2 u = \lambda u$ with boundary data (1.12) may change sign [12].

Remark 1.2. Let $V = \text{span}\{\phi_1\}$ be the eigenspace associated with λ_1 , where $\|\phi_1\|_{2,p} = 1$. Taking a subspace $W \subset W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega)$ complementing V , that is, $W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega) = V \oplus W$, there exists $\hat{\lambda} > \lambda_1$ with

$$\int_{\Omega} |\Delta u|^p dx \geq \hat{\lambda} \int_{\Omega} |u|^p dx \quad (1.13)$$

for each $u \in W$ (in case $p = 2$, one may take $\hat{\lambda} = \lambda_2$).

We may now assume the following conditions:

(\mathcal{H}_1) there exist $R > 0$ and $\bar{\lambda} \in]\lambda_1, \hat{\lambda}[$ such that

$$|s| \leq R \implies \lambda_1 |s|^p \leq pG(x, s) \leq \bar{\lambda} |s|^p, \quad (1.14)$$

for a.e. $x \in \Omega$ and each $s \in \mathbb{R}$;

(\mathcal{H}_2) there exist $\vartheta > p$ and $M > 0$ such that

$$|s| \geq M \implies 0 < \vartheta G(x, s) \leq s g(x, s), \quad (1.15)$$

for a.e. $x \in \Omega$ and each $s \in \mathbb{R}$.

Assumption (\mathcal{H}_1) corresponds to a resonance condition around the origin while (\mathcal{H}_2) is the standard condition of Ambrosetti-Rabinowitz type.

THEOREM 1.3. *Assume that conditions (\mathcal{H}_1) and (\mathcal{H}_2) hold. Then problem (1.1) with boundary conditions (1.2) admits a nontrivial solution in $W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega)$.*

Now replace (\mathcal{H}_2) with a nonresonance condition at $+\infty$.

THEOREM 1.4. *Assume that condition (\mathcal{H}_1) holds and that for a.e. $x \in \Omega$*

$$\lim_{|s| \rightarrow +\infty} \frac{pG(x, s)}{|s|^p} < \lambda_1. \quad (1.16)$$

Then problem (1.1) with boundary conditions (1.2) admits two nontrivial solutions in $W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega)$.

We use variational methods to prove Theorems 1.3 and 1.4. Usually, one uses a minimax type argument of mountain pass type to prove the existence of solutions of equations with a variational structure. However, it seems difficult to use minimax theorems in our situation. Thus we will adopt an approach based on Morse theory. Notice that there were a few works using Morse theory to treat p -Laplacian problems with Dirichlet boundary conditions (see [9] and the references therein). Moreover, to the authors' knowledge, (1.1) has a very poor literature; the only papers in which a p -harmonic equation is mentioned are [1, Section 8] and [4].

The existence of multiple solutions depends mainly on the behaviour of $G(x, s)$ near 0 and at $+\infty$. Without the above resonant or nonresonant conditions to obtain multiple solutions seems hard even in the semilinear case $p = 2$.

Remark 1.5. Arguing as in [9], it is possible to prove [Theorem 1.4](#) by replacing assumption [\(1.16\)](#) with the following conditions:

$$\lim_{|s| \rightarrow +\infty} \frac{pG(x, s)}{|s|^p} = \lambda_1, \quad \lim_{|s| \rightarrow +\infty} \{g(x, s)s - pG(x, s)\} = +\infty \quad (1.17)$$

for a.e. $x \in \Omega$ (resonance condition at $+\infty$).

Remark 1.6. The existence of solutions $u \in W_0^{2,p}(\Omega)$ of the quasilinear problem

$$\begin{aligned} \Delta(|\Delta u|^{p-2} \Delta u) &= g(x, u) && \text{in } \Omega, \\ u &= \nabla u = 0 && \text{on } \partial\Omega \end{aligned} \quad (1.18)$$

under the previous assumptions (\mathcal{H}_j) is, to our knowledge, an open problem.

2. Proofs of Theorems 1.3 and 1.4

In this section, we give the proof of our main results. It is readily seen that

$$\|u\|_{2,p} = \left(\int_{\Omega} |\Delta u|^p dx \right)^{1/p} \quad (2.1)$$

is an equivalent norm of the standard space norm of $W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega)$. For Φ a continuously Fréchet differentiable map, let Φ' denote its Fréchet derivative.

LEMMA 2.1. *The functional Φ satisfies the Palais-Smale condition.*

Proof. Let $(u_h) \subset W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega)$ be such that $|\Phi(u_h)| \leq B$, for some $B > 0$ and $\Phi'(u_h) \rightarrow 0$ as $h \rightarrow +\infty$. Let $d = \sup_{h \geq 0} \Phi(u_h)$. Then we have

$$\begin{aligned} \vartheta d + \|u_h\|_{2,p} &\geq \vartheta \Phi(u_h) - \langle \Phi'(u_h), u_h \rangle \\ &= \left(\frac{\vartheta}{p} - 1 \right) \|u_h\|_{2,p}^p - \int_{\{|u_h| \geq M\}} [\vartheta G(x, u_h) - g(x, u_h) u_h] dx \\ &\quad - \int_{\{|u_h| \leq M\}} [\vartheta G(x, u_h) - g(x, u_h) u_h] dx \\ &\geq \left(\frac{\vartheta}{p} - 1 \right) \|u_h\|_{2,p}^p - \int_{\{|u_h| \leq M\}} [\vartheta G(x, u_h) - g(x, u_h) u_h] dx \\ &\geq \left(\frac{\vartheta}{p} - 1 \right) \|u_h\|_{2,p}^p - D, \end{aligned} \quad (2.2)$$

for some $D \in \mathbb{R}$. Thus (u_h) is bounded and, up to a subsequence, we may assume that $u_h \rightharpoonup u$ is, for some u , in $W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega)$. Since the embedding $W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega) \hookrightarrow L^q(\Omega)$ is compact, then a standard argument shows that $u_h \rightarrow u$ strongly and the proof is complete. \square

Now recall the notion of “Local Linking,” which was initially introduced by Liu and Li [8] and has been used in a vast amount of literature (cf. [2, 5, 6, 11]).

Definition 2.2. Let E be a real Banach space such that $E = V \oplus W$, where V and W are closed subspaces of E . Let $\Phi : E \rightarrow \mathbb{R}$ be a C^1 -functional. We say that Φ has a local linking near the origin 0 (with respect to the decomposition $E = V \oplus W$), if there exists $\varrho > 0$ such that

$$\begin{aligned} u \in V : \|u\| \leq \varrho &\implies \Phi(u) \leq 0, \\ u \in W : 0 < \|u\| \leq \varrho &\implies \Phi(u) > 0. \end{aligned} \tag{2.3}$$

We now show that our functional Φ has a local linking near the origin with respect to the space decomposition $W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega) = V \oplus W$, according to **Remark 1.2**.

LEMMA 2.3. *There exists $\varrho > 0$ such that conditions (2.3) hold with respect to the decomposition $W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega) = V \oplus W$.*

Proof. For $u \in V$, the condition $\|u\|_{2,p} \leq \varrho$ implies $u(x) \leq R$ for a.e. $x \in \Omega$ if $\varrho > 0$ is small enough, being $R > 0$ as in assumption (\mathcal{H}_1) . Thus for $u \in V$,

$$\begin{aligned} \Phi(u) &= \frac{1}{p} \int_{\Omega} |\Delta u|^p dx - \int_{\Omega} G(x, u) dx \\ &= \frac{\lambda_1}{p} \int_{\Omega} |u|^p dx - \int_{\Omega} G(x, u) dx = \int_{\{|u| \leq R\}} \left[\frac{\lambda_1}{p} |u|^p - G(x, u) \right] dx \leq 0 \end{aligned} \tag{2.4}$$

provided that $\|u\|_{2,p} \leq \varrho$ and ϱ is small.

To prove the second assertion, take $u \in W$. In view of (1.3) and (1.13) we have

$$\begin{aligned} \Phi(u) &= \frac{1}{p} \int_{\Omega} |\Delta u|^p dx - \int_{\Omega} G(x, u) dx \\ &= \frac{1}{p} \int_{\Omega} (|\Delta u|^p - \bar{\lambda} |u|^p) dx \\ &\quad - \left(\int_{\{|u| \leq R\}} + \int_{\{|u| \geq R\}} \right) \left(G(x, u) - \frac{\bar{\lambda}}{p} |u|^p \right) dx \\ &\geq \frac{1}{p} \left(1 - \frac{\bar{\lambda}}{\hat{\lambda}} \right) \|u\|_{2,p}^p - c \int_{\Omega} |u|^s dx \geq \frac{1}{p} \left(1 - \frac{\bar{\lambda}}{\hat{\lambda}} \right) \|u\|_{2,p}^p - C \|u\|_{2,p}^s, \end{aligned} \tag{2.5}$$

where $p < s \leq p_*$ and c, C are positive constants. Since $s > p$, it follows that $\Phi(u) > 0$ for $\varrho > 0$ sufficiently small. \square

Assume that u is an isolated critical point of Φ such that $\Phi(u) = c$. We define the *critical group* of Φ at u by setting for each $q \in \mathbb{Z}$

$$C_q(\Phi, u) = H_q(\Phi_c, \Phi_c \setminus \{u\}), \tag{2.6}$$

being $H_q(X, Y)$ the q th homology group of the topological pair (X, Y) over the ring \mathbb{Z} and Φ_c the c -sublevel of Φ . For the detail of Morse theory and critical groups, we refer the reader to [3].

Since $\dim V = 1 < +\infty$, by combining [Lemma 2.3](#) and [\[7, Theorem 2.1\]](#), we obtain the following result.

LEMMA 2.4. *The point 0 is a critical point of Φ and $C_1(\Phi, 0) \neq \{0\}$.*

We now investigate the behavior of Φ near infinity.

LEMMA 2.5. *There exists a constant $A > 0$ such that*

$$a < -A \implies \Phi_a \simeq S^\infty, \quad (2.7)$$

where $S^\infty = \{u \in W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega) : \|u\|_{2,p} = 1\}$.

Proof. By integrating inequality [\(1.15\)](#), we obtain a constant $C_1 > 0$ with

$$|s| \geq M \implies G(x, s) \geq C_1 |s|^\vartheta \quad (2.8)$$

a.e. in Ω and for each $s \in \mathbb{R}$. Thus, for $u \in S^\infty$, we have $\Phi(tu) \rightarrow -\infty$, as t goes to $+\infty$. Set

$$A = \left(1 + \frac{1}{p}\right) M \mathcal{L}^n(\Omega) \max_{\bar{\Omega} \times [-M, M]} |g(x, s)| + 1, \quad (2.9)$$

being \mathcal{L}^n the Lebesgue measure. As in the proof of [\[10, Lemma 2.4\]](#) we obtain

$$\begin{aligned} & \int_{\Omega} G(x, u) dx - \frac{1}{p} \int_{\Omega} g(x, u) u dx \\ & \leq \left(\frac{1}{\vartheta} - \frac{1}{p} \right) \int_{\{|u| \geq M\}} g(x, u) u dx + A - 1. \end{aligned} \quad (2.10)$$

For $a < -A$ and

$$\Phi(tu) = \frac{|t|^p}{p} - \int_{\Omega} G(x, tu) dx \leq a \quad (u \in S^\infty), \quad (2.11)$$

in view of [\(2.8\)](#) and [\(2.10\)](#), arguing as in the proof of [\[10, Lemma 2.4\]](#),

$$\frac{d}{dt} \Phi(tu) < 0. \quad (2.12)$$

By the implicit function theorem, there is a unique $T \in C(S^\infty, \mathbb{R})$ such that

$$\forall u \in S^\infty, \quad \Phi(T(u)u) = a. \quad (2.13)$$

For $u \neq 0$, set $\tilde{T}(u) = (1/\|u\|_{2,p})T(u/\|u\|_{2,p})$. Then $\tilde{T} \in C(W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega) \setminus \{0\}, \mathbb{R})$ and

$$\forall u \in W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega) \setminus \{0\}, \quad \Phi(\tilde{T}(u)u) = a. \quad (2.14)$$

We define now a functional $\hat{T} : W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega) \setminus \{0\} \rightarrow \mathbb{R}$ by setting

$$\hat{T}(u) = \begin{cases} \tilde{T}(u) & \text{if } \Phi(u) \geq a, \\ 1 & \text{if } \Phi(u) \leq a. \end{cases} \quad (2.15)$$

Since $\Phi(u) = a$ implies $\tilde{T}(u) = 1$, we conclude that

$$\hat{T} \in C(W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega) \setminus \{0\}, \mathbb{R}). \quad (2.16)$$

Finally, let $\eta : [0, 1] \times W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega) \setminus \{0\} \rightarrow W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega) \setminus \{0\}$,

$$\eta(s, u) = (1-s)u + s\hat{T}(u)u. \quad (2.17)$$

It results that η is a strong deformation retract from $W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega) \setminus \{0\}$ to Φ_a . Thus $\Phi_a \simeq W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega) \setminus \{0\} \simeq S^\infty$. \square

Remark 2.6. A result similar to [Lemma 2.5](#) has been proved for the Laplacian $-\Delta$ in [\[3, 14\]](#), under the additional conditions

$$g \in C^1(\Omega \times \mathbb{R}, \mathbb{R}), \quad g_t(x, 0) = \frac{\partial g(x, t)}{\partial t} \Big|_{t=0} = 0. \quad (2.18)$$

We recall the following topological result due to Perera [\[11\]](#).

LEMMA 2.7. *Let $Y \subset B \subset A \subset X$ be topological spaces and $q \in \mathbb{Z}$. If*

$$H_q(A, B) \neq \{0\}, \quad H_q(X, Y) = \{0\}, \quad (2.19)$$

then it results that

$$H_{q+1}(X, A) \neq \{0\} \quad \text{or} \quad H_{q-1}(B, Y) \neq \{0\}. \quad (2.20)$$

Proof of Theorem 1.3. By [Lemma 2.1](#), Φ satisfies the Palais-Smale condition. Note that $\Phi(0) = 0$, by [\[3, Chapter I, Theorem 4.2\]](#), there exists $\varepsilon > 0$ with

$$H_1(\Phi_\varepsilon, \Phi_{-\varepsilon}) = C_1(\Phi, 0) \neq \{0\}. \quad (2.21)$$

If A is as in [Lemma 2.5](#), for $a < -A$ we have $\Phi_a \simeq S^\infty$, which yields

$$H_1(W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega), \Phi_a) = H_1(W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega), S^\infty) = \{0\}. \quad (2.22)$$

Therefore, being $\Phi_a \subset \Phi_{-\varepsilon} \subset \Phi_\varepsilon$, [Lemma 2.7](#) yields

$$H_2(W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega), \Phi_\varepsilon) \neq \{0\} \quad \text{or} \quad H_0(\Phi_{-\varepsilon}, \Phi_a) \neq \{0\}. \quad (2.23)$$

It follows that Φ has a critical point u for which

$$\Phi(u) > \varepsilon \quad \text{or} \quad -\varepsilon > \Phi(u) > a. \quad (2.24)$$

Therefore, $u \neq 0$ and [\(1.1\)](#), [\(1.2\)](#) possess a nontrivial solution. \square

Recall from [9] the following three-critical point theorem.

LEMMA 2.8. *Let X be a real Banach space and let $\Phi \in C^1(X, \mathbb{R})$ be bounded from below and satisfying the Palais-Smale condition. Assume that Φ has a critical point u which is homologically nontrivial, that is, $C_j(\Phi, u) \neq \{0\}$ for some j , and it is not a minimizer for Φ . Then Φ admits at least three critical points.*

Proof of Theorem 1.4. By Lemma 2.8, taking into account Lemma 2.4, it suffices to show that Φ is bounded from below. Indeed, by (1.16) there exist $\varepsilon > 0$ small and $C > 0$ such that

$$G(x, s) \leq \frac{\lambda_1 - \varepsilon}{p} |s|^p + C \quad (2.25)$$

for a.e. $x \in \Omega$ and each $s \in \mathbb{R}$. This, by (1.11), immediately yields

$$\begin{aligned} \Phi(u) &\geq \frac{1}{p} \|u\|_{2,p}^p - \frac{1}{p} (\lambda_1 - \varepsilon) \|u\|_p^p - C \mathcal{L}^n(\Omega) \\ &\geq \frac{1}{p} \left(1 - \frac{\lambda_1 - \varepsilon}{\lambda_1} \right) \|u\|_{2,p}^p - C \mathcal{L}^n(\Omega) \longrightarrow +\infty \end{aligned} \quad (2.26)$$

as $\|u\|_{2,p} \rightarrow +\infty$. Then Φ is coercive and satisfies the Palais-Smale condition. In particular Lemma 2.8 provides the existence of at least two nontrivial critical points of Φ . \square

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