INSCRIBING CLOSED NON-σ-LOWER POROUS SETS INTO SUSLIN NON-σ-LOWER POROUS SETS

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The main aim of this paper is to prove that every non-σ-lower porous Suslin set in a topologically complete metric space contains a closed non-σ-lower porous subset. In fact, we prove a general result of this type on "abstract porosities." This general theorem is also applied to ball small sets in Hilbert spaces and to σ-cone-supported sets in separable Banach spaces.

1. Introduction

This paper is a continuation of the work done in [9]. We are interested in the following question within the context of σ-ideals of σ-porous type.

Let X be a metric space and let $\mathcal{I}$ be a σ-ideal of subsets of X. Let $S \subset X$ be a Suslin set with $S \notin \mathcal{I}$. Does there exist a closed set $F \subset S$ which is not in $\mathcal{I}$?

The answer is positive provided that X is locally compact and $\mathcal{I}$ is a σ-ideal of σ-**P**-porous sets, where **P** is a porosity-like relation satisfying some additional conditions (see the definitions below, and for the precise statement, see [9]). In the case of the σ-ideal of ordinary (i.e., upper) σ-porous sets, which satisfies the assumptions of the above-mentioned theorem in any locally compact metric space, even more is true: X can be any topologically complete metric space (see [8]). The proofs are not easy; they use either some amount of descriptive set theory (see [9]) or a quite complicated construction (see [8]).

In this paper, we deal with σ-ideals of σ-**P**-porous sets again, but these σ-ideals are supposed to be generated by closed **P**-porous sets, that is, every σ-**P**-porous set is covered by countably many closed **P**-porous sets. Note that this property does not hold for ordinary σ-porous sets but does hold for σ-lower porous sets. Although we will also work in nonseparable spaces, it turns out that the situation is much simpler than in [9]. Under a simple additional condition on the porosity-like relation **P**, we prove that every such σ-ideal has the property that every non-σ-**P**-porous Suslin subset of a topologically complete metric space X contains a closed non-σ-**P**-porous subset. As the main tool, we use a nonseparable version of Solecki's theorem proved in [2].

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The general result will be applied to the σ-ideals of σ-lower porous sets, of σ-cone-supported sets, and of ball small sets.

2. The general result

We start with notations and definitions. Let (X,ρ) be a metric space. Then the open ball with center $x \in X$ and radius $r > 0$ is denoted by $B(x,r)$. We will use the following terminology from [7, 9]. We say that $\mathbf{R}$ is a *point-set relation on* X if it is a relation between points of X and subsets of X. Thus a point-set relation $\mathbf{R}$ is a subset of $X \times 2^X$. The symbol $\mathbf{R}(x,A)$, where $x \in X$ and $A \subset X$, means that $(x,A) \in \mathbf{R}$, that is, $\mathbf{R}$ holds for the pair (x,A).

Let $\mathbf{R}$ be a point-set relation on X. If $A \subset X$ and $B \subset X$, then $\mathbf{R}(A,B) \overset{\text{def}}{\iff} \forall a \in A : \mathbf{R}(a,B)$. The point-set relation $\neg\mathbf{R}$ on X is defined by $(\neg\mathbf{R})(x,A) \overset{\text{def}}{\iff} \neg(\mathbf{R}(x,A))$.

We consider the following properties of a point-set relation $\mathbf{R}$ on X.

(A1) If $A \subset B \subset X$, $x \in X$, and $\mathbf{R}(x,B)$, then $\mathbf{R}(x,A)$.

(A2) $\mathbf{R}(x,A)$ if and only if there is $r > 0$ such that $\mathbf{R}(x,A \cap B(x,r))$.

(A3) $\mathbf{R}(x,A)$ if and only if $\mathbf{R}(x,\overline{A})$.

We say that a point-set relation $\mathbf{P}$ on X is a *porosity-like relation* if $\mathbf{P}$ satisfies the "axioms" (A1)–(A3).

Let $\mathbf{P}$ be a porosity-like relation on X. We say that $A \subset X$ is

(i) $\mathbf{P}$-*porous at* $x \in X$ if $\mathbf{P}(x,A)$,

(ii) $\mathbf{P}$-*porous* if $\mathbf{P}(x,A)$ for every $x \in A$,

(iii) σ-$\mathbf{P}$-*porous* if A is a countable union of $\mathbf{P}$-porous sets.

If $\mathbf{P}$ is a porosity-like relation on X and $A \subset X$, then the set of all points of A, at which A is not $\mathbf{P}$-porous, is denoted by $\mathrm{N}(\mathbf{P},A)$.

The proof of our result is based on the following nonseparable version (see [2, Corollary 3.6 and Remark 3.7]) of Solecki's theorem (see [3]). We need the following definitions to formulate it.

Let $\mathcal{A}$ be a system of subsets of a metric space X. We say that $\mathcal{A}$ is *weakly locally determined* if $A \subset X$ belongs to $\mathcal{A}$ whenever for each $x \in X$ there exists a, not necessarily open, neighbourhood U of x such that $U \cap A \in \mathcal{A}$.

Let $\mathcal{F}$ be a family of closed subsets of a metric space X. We say that $\mathcal{F}$ is *hereditary* if for all sets F_1, F_2 with $F_1 \subset F_2$, $F_2 \in \mathcal{F}$, we have $F_1 \in \mathcal{F}$.

Proposition 2.1 (see [2]). *Let X be a topologically complete metric space. Let $\mathcal{F}$ be a hereditary weakly locally determined system of closed sets. Then each Suslin subset of X is either covered by countably many elements of $\mathcal{F}$ or else contains a $\mathbf{G}_\delta$ set H such that $H \cap G$ cannot be covered by countably many elements of $\mathcal{F}$, whenever G is open and $G \cap H \neq \emptyset$.*

Definition 2.2. Let X be a metric space and let $\mathbf{P}$ be a porosity-like relation on X. It is said that $\mathbf{P}$ has property $(\star)$ if the following condition is satisfied.

$(\star)$ If $H \subset X$, $x \in H'$, and H is not $\mathbf{P}$-porous at x, then there exists $J \subset H$ such that $J' = \{x\}$ and J is not $\mathbf{P}$-porous at x.

The symbol H' stands for the set of all points of accumulation of H.

Now we can formulate our abstract theorem.

Theorem 2.3. *Let X be a topologically complete metric space and let $\mathbf{P}$ be a porosity-like relation on X such that $\mathbf{P}$ satisfies $(\star)$, and each σ-$\mathbf{P}$-porous set is covered by countably many closed $\mathbf{P}$-porous sets. If $S \subset X$ is a Suslin non-σ-$\mathbf{P}$-porous set, then there exists a closed non-σ-$\mathbf{P}$-porous set $F \subset S$.*

The next lemma immediately follows by a Baire category argument.

Lemma 2.4. *Let X and $\mathbf{P}$ be as in Theorem 2.3. Let $F \subset X$ be a closed nonempty set such that $\mathrm{N}(\mathbf{P},F)$ is dense in F. Then F is not σ-$\mathbf{P}$-porous.*

Proof of Theorem 2.3. We denote the σ-ideal of all σ-$\mathbf{P}$-porous sets by $\mathcal{I}$.

The system of all closed $\mathbf{P}$-porous sets is clearly hereditary and weakly locally determined by (A1) and (A2). According to Proposition 2.1, we may and do assume that S is a $\mathbf{G}_\delta$ set and $S \cap G \notin \mathcal{I}$ for every open $G \subset X$ intersecting S. If there is $x \in S \setminus S'$, then $\{x\} \notin \mathcal{I}$. In this case, $F := \{x\}$ can serve as the set for which we are looking. From now on, we assume that $S \subset S'$. Let $S = \bigcap_{n=1}^{\infty} G_n$, where $\{G_n\}_{n=1}^{\infty}$ is a decreasing sequence of open sets. We will construct a sequence $\{F_n\}_{n=0}^{\infty}$ of closed sets and a decreasing sequence $\{H_n\}_{n=1}^{\infty}$ of open sets such that $F_0 = \varnothing$ and for every $n \in \mathbb{N}$, we have

(a) $\varnothing \neq F_n \subset \mathrm{N}(\mathbf{P},S)$,
(b) $F_n' = F_{n-1}$,
(c) $F_n \subset H_n \subset \overline{H_n} \subset G_n$,
(d) $(\neg\mathbf{P})(F_{n-1},F_n)$.

We proceed by induction over n. Since $S \notin \mathcal{I}$, we can choose $x \in \mathrm{N}(\mathbf{P},S)$. We put $F_1 = \{x\}$. We easily find an open set H_1 such that $x \in H_1$ and $\overline{H_1} \subset G_1$. The sets F_1 and H_1 satisfy (a)–(d) for $n = 1$.

Assume that we have constructed $F_1,\dots,F_m$ and $H_1,\dots,H_m$ such that (a)–(d) hold for $n = 1,\dots,m$. We find an open set H_{m+1} with $F_m \subset H_{m+1} \subset \overline{H_{m+1}} \subset G_{m+1} \cap H_m$. The set $F_m \setminus F_m'$ is discrete in $X \setminus F_m'$, that is, for every $y \in X \setminus F_m'$, there exists $r > 0$ such that $B(y,r) \cap (F_m \setminus F_m')$ contains at most one point. It is well known and easy to prove that, for each $z \in F_m \setminus F_m'$, we can choose $r_z > 0$ such that $\mathcal{B} = (B(z,r_z))_{z \in F_m \setminus F_m'}$ is discrete in $X \setminus F_m'$, that is, for every $y \in X \setminus F_m'$, there exists $s > 0$ such that, for at most one $z \in F_m \setminus F_m'$, $B(y,s)$ intersects $B(z,r_z)$.

Since $S \cap G \notin \mathcal{I}$ for every open G intersecting S, we have that $\mathrm{N}(\mathbf{P},S)$ is dense in S. According to this, (A3), and (a), we have $F_m \subset \mathrm{N}(\mathbf{P},\mathrm{N}(\mathbf{P},S))$. Thus using the condition $(\star)$ and (A2), we find for every $z \in F_m \setminus F_m'$ a set J_z such that $J_z \subset B(z,r_z) \cap H_{m+1} \cap \mathrm{N}(\mathbf{P},S)$, $(\neg\mathbf{P})(z,J_z)$, and $J_z' = \{z\}$.

We put $F_{m+1} = F_m \cup \bigcup\{J_z;\ z \in F_m \setminus F_m'\}$. Clearly, $F_{m+1} \subset \mathrm{N}(\mathbf{P},S)$ and $F_{m+1} \subset H_{m+1}$. It is easy to see that $F_{m+1}' = F_m$; in particular, F_{m+1} is closed.

Let $x \in F_m$. We distinguish two possibilities. If $x \in F_m' = F_{m-1}$, then $(\neg\mathbf{P})(x,F_m)$ by the induction hypothesis, and so $(\neg\mathbf{P})(x,F_{m+1})$ by (A1). If $x \in F_m \setminus F_m'$, then $(\neg\mathbf{P})(x,J_x)$ and we also have $(\neg\mathbf{P})(x,F_{m+1})$. We get $(\neg\mathbf{P})(F_m,F_{m+1})$. Thus the sets F_{m+1} and H_{m+1} satisfy (a)–(d) for $n = m+1$ and the construction of our sequences is finished.

The desired set F is defined by $F = \overline{\bigcup_{n=1}^{\infty} F_n}$. Using (c) and the monotonicity of the H_n's, we get $F \subset S$. We have $(\neg\mathbf{P})(\bigcup_{n=1}^{\infty} F_n, F)$ by (d). The set $\bigcup_{n=1}^{\infty} F_n$ is dense in F. Hence $F \notin \mathcal{I}$, by Lemma 2.4. $\square$

3. Applications

We will apply Theorem 2.3 to the σ-ideal of σ-lower porous sets (in a topologically complete metric space) and to two of its subsystems: to the σ-ideal of σ-cone-supported sets (in a separable Banach space) and to the σ-ideal of ball small sets (in an arbitrary Hilbert space).

Note that σ-lower porous sets (called frequently simply "σ-porous sets" and sometimes "σ-very porous sets") were applied in a number of articles on exceptional sets in (sometimes also nonseparable) Banach spaces (cf. [6]). In [6], information on σ-cone-supported and ball small sets can also be found.

To verify condition ($\star$) in concrete cases, we will apply the following easy lemma.

Lemma 3.1. *Let $g : [0,\infty) \to [0,\infty)$ be a continuous increasing function with $g(0) = 0$. Let (X,ρ) be a metric space, $H \subset X$, and $a \in H'$. Then there exists $J \subset H \setminus \{a\}$, such that $J' = \{a\}$, and for each $x \in H \setminus \{a\}$, there exists $x^* \in J$ such that $g(\rho(x,x^*)) < \min(\rho(x,a), \rho(x^*,a))$.*

Proof. Let $M_1 := \{x \in X;\ 1 \le \rho(x,a)\}$ and $M_n := \{x \in X;\ 1/n \le \rho(x,a) < 1/(n-1)\}$ for $n = 2,3,\ldots$. For each natural n, choose $\varepsilon_n > 0$ such that $g(\varepsilon_n) < 1/n$ and in $H \cap M_n$, find a maximal ε_n-discrete subset D_n ($\rho(u,v) \ge \varepsilon_n$ for each $u, v \in D_n$, $u \ne v$). Put $J := \bigcup_{n=1}^{\infty} D_n$. Clearly, $J \subset H \setminus \{a\}$ and $J' = \{a\}$. Let $x \in H \setminus \{a\}$ be given. Find $n \in \mathbb{N}$ with $x \in M_n$. By maximality of D_n, we can choose $x^* \in D_n \subset J$ with $\rho(x,x^*) < \varepsilon_n$. Consequently,

$$g(\rho(x,x^*)) < g(\varepsilon_n) < \frac{1}{n} \le \min(\rho(x,a),\rho(x^*,a)). \tag{3.1}$$

□

3.1. σ-lower porous sets

Definition 3.2. Let (X,ρ) be a metric space. It is said that $A \subset X$ is *lower porous at $x \in X$* if there exist $c > 0$ and $r_0 > 0$ such that for every $r \in (0,r_0)$, there exists $y \in B(x,r)$ with $B(y,cr) \subset B(x,r) \setminus A$. The corresponding porosity-like relation is denoted by $\mathbf{P_l}$, and σ-$\mathbf{P_l}$-porous sets are called *σ-lower porous*.

It is a well known and an easy fact that the σ-ideal $\mathcal{I}_l$ of all σ-lower porous sets is generated by closed $\mathbf{P_l}$-porous sets (see, e.g., [6, Proposition 2.5]). The proof of the following lemma is also easy.

Lemma 3.3. *Let X be a metric space. Then $\mathbf{P_l}$ has property ($\star$).*

Proof. Let $x \in \mathrm{N}(\mathbf{P},H) \cap H'$. Put $g(h) := \sqrt{h}$ (then $h = o(g(h))$, $h \to 0+$) and find $J \subset H$ by Lemma 3.1. Then $J' = \{x\}$. We will prove $(\neg\mathbf{P_l})(x,J)$.

Suppose on the contrary that J is lower porous at x. Then there exist $c > 0$ and $r_0 > 0$ such that for each $0 < r < r_0$, there exists $y \in X$ with $B(y,cr) \subset B(x,r) \setminus J$. We can clearly choose $r_1 > 0$ such that $g(h) > 2h/c$ for each $0 < h < r_1$. Put $\tilde{r} := \min(r_0,r_1)$, $\tilde{c} := c/2$, and consider an arbitrary $0 < r < \tilde{r}$. Choose $y \in X$ such that $B(y,cr) \subset B(x,r) \setminus J$. To obtain a contradiction with $x \in \mathrm{N}(\mathbf{P_l},H)$, it is sufficient to show that

$$B(y,\tilde{c}r) \cap H = \emptyset. \tag{3.2}$$

Suppose that it is not the case and choose $z \in B(y,\tilde{c}r) \cap H$. By the choice of J, we can find $z^* \in J$ such that $g(\rho(z,z^*)) < \rho(z,x) < r < r_1$. Since $\tilde{c} < c$, we have $z \neq z^*$ and the definition of r_1 gives $g(\rho(z,z^*)) > 2\rho(z,z^*)/c$. Consequently, $\rho(z,z^*) < cr/2$, which implies that $z^* \in B(y,cr) \cap J$. This is a contradiction which proves (3.2). □

Theorem 2.3 thus implies the following result.

Corollary 3.4. *Let X be a topologically complete metric space and let $S \subset X$ be a Suslin set which is not σ-lower porous. Then there exists a closed $F \subset S$ which is not σ-lower porous.*

Remark 3.5. We say that $A \subset \mathbf{R}$ is *lower symmetrically porous at* $x \in \mathbf{R}$ if there exist $r_0 > 0$ and $c > 0$ such that for each $0 < r < r_0$, there exist $h > 0$ and $t \geq 0$ such that $h/r > c$, $t + h \leq r$, $(x+t, x+t+h) \cap A = \emptyset$, and $(x-t-h, x-t) \cap A = \emptyset$. The notions of a *lower symmetrically porous set* and a *σ-lower symmetrically porous set* are defined in the obvious way.

Proceeding quite similarly as above, we can easily obtain that *each analytic set $S \subset \mathbf{R}$ which is not σ-lower symmetrically porous contains a closed set which is not σ-lower symmetrically porous.*

3.2. Cone-supported sets

Definition 3.6. If X is a Banach space, $v \in X$, $\|v\| = 1$, and $0 < c < 1$, then define the cone $A(v,c) := \bigcup_{\lambda>0} \lambda \cdot B(v,c)$. Define the (clearly porosity-like) point-set relation $\mathbf{P_s}$ as follows: $\mathbf{P_s}(x,M)$ if there exist $r > 0$ and a cone $A(v,c)$ such that $M \cap (x + A(v,c)) \cap B(x,r) = \emptyset$. Sets which are $\mathbf{P_s}$-porous (σ-$\mathbf{P_s}$-porous) are called *cone supported* (*σ-cone supported*).

If X is separable, it is easy to prove (see [4, Lemma 1], cf. [6]) that $M \subset X$ is σ-cone supported (i.e., σ-$\mathbf{P_s}$-porous) if and only if M can be covered by countably many Lipschitz hypersurfaces. Since each Lipschitz hypersurface is clearly a closed $\mathbf{P_s}$-porous set, every σ-$\mathbf{P_s}$-porous set is covered by countably many closed $\mathbf{P_s}$-porous sets.

Lemma 3.7. *Let X be a Banach space. Then $\mathbf{P_s}$ has property ($\star$).*

Proof. Let $x \in \mathrm{N}(\mathbf{P_s},H) \cap H'$. Put $g(h) := \sqrt{h}$ and find $J \subset H$ by Lemma 3.1. Then $J' = \{x\}$. We will prove $(\neg\mathbf{P_s})(x,J)$. We can and will suppose that $x = 0$.

Suppose on the contrary that $\mathbf{P_s}(0,J)$. Then there exist $v \in X$, with $\|v\| = 1$, $1 > c > 0$, and $r > 0$ such that $J \cap A(v,c) \cap B(0,r) = \emptyset$. We can suppose that $r < c/4$. To obtain a contradiction with $0 \in \mathrm{N}(\mathbf{P_s},H)$, it is sufficient to show that

$$H \cap A\left(v,\frac{c}{2}\right) \cap B\left(0,\frac{r}{2}\right) = \emptyset. \tag{3.3}$$

Suppose that this is not the case and choose $z \in H \cap A(v,c/2) \cap B(0,r/2)$. By the choice of J, we can find $z^* \in J$ such that $\|z - z^*\| \leq \|z\|^2 < \min(r/2, c/4 \cdot \|z\|)$. Thus clearly $z^* \in B(0,r)$. Choose $\lambda > 0$ with $\|\lambda z - v\| < c/2$. Then

$$||\lambda z^* - v|| \leq \frac{c}{2} + \lambda||z - z^*|| \leq \frac{c}{2} + \|\lambda z\| \cdot \frac{c}{4} < \frac{c}{2} + \left(1 + \frac{c}{2}\right) \cdot \frac{c}{4} < c, \tag{3.4}$$

and thus $z^* \in A(v,c) \cap B(0,r)$. This is a contradiction which proves (3.3). □

Theorem 2.3 thus implies the following result.

Corollary 3.8. *Let X be a separable Banach space and let $S \subset X$ be an analytic set which cannot be covered by countably many Lipschitz hypersurfaces. Then there exists a closed set $F \subset S$ which cannot be covered by countably many Lipschitz hypersurfaces.*

3.3. Ball small sets

Definition 3.9. Let X be a Banach space and let $r > 0$. It is said that $A \subset X$ is *r-ball porous* at a point $x \in A$ if for each $\varepsilon \in (0,r)$, there exists $y \in X$ such that $\|x - y\| = r$ and $B(y, r - \varepsilon) \cap A = \emptyset$. A set $A \subset X$ is called *r-ball porous* if it is r-ball porous at each $x \in A$. It is said that $A \subset X$ is *ball small* if it can be written in the form $A = \bigcup_{n=1}^{\infty} A_n$, where each A_n is r_n-ball porous for some $r_n > 0$.

Using the obvious fact that $B(z, \|z - x\| - \varepsilon) \subset B(y, \rho - \varepsilon)$ whenever $\|y - x\| = \rho > 0$, z lies on the segment xy, and $\|z - x\| > \varepsilon > 0$, it is easy to verify the following facts.

(i) If A is r-ball porous at a and $0 < r^* < r$, then A is r^*-ball porous at a.

(ii) If A is r-ball porous, then $\overline{A}$ is $r/2$-ball porous.

For $A \subset X$ and $x \in X$, we will write $\mathbf{P_b}(x, A)$ if A is r-ball porous at x for some $r > 0$.

Using (i), it is easy to see that $\mathbf{P_b}$ is a porosity-like relation on X and that the σ-ideal $\mathcal{I}_b$ of all ball small sets coincides with the system of all σ-$\mathbf{P_b}$-porous sets.

By (ii), we easily obtain that $\mathcal{I}_b$ is generated by closed $\mathbf{P_b}$-porous sets.

The proof of the following lemma is not difficult but slightly technical.

Lemma 3.10. *Let X be a Hilbert space. Then $\mathbf{P_b}$ has property ($\star$).*

Proof (Sketch). First, observe that an elementary (two-dimensional) computation gives the following fact.

(F) If b, v, x, x^* are points of X, $\|v\| = 1$, $0 < \rho < 1/10$, $x \in B(b + \rho/2 \cdot v, \rho/2)$, and $\|x^* - x\| \le 4\|b - x\|^2$, then $x^* \in B(b + \rho v, \rho)$.

Now let $H \subset X$ and $a \in \mathrm{N}(\mathbf{P_b}, H) \cap H'$. Put $g(h) := \sqrt{h}$ and find $J \subset H$ by Lemma 3.1. Then $J' = \{a\}$. We will prove $(\neg\mathbf{P_b})(a, J)$. Suppose to the contrary that J is r-ball porous at a for some $r > 0$. By (i), we can suppose that $r < 1/10$. Then for each $0 < \varepsilon < r/4$, there exists $v \in X$ with $\|v\| = 1$ such that $B(a + rv, r - \varepsilon) \cap J = \emptyset$. It is sufficient to prove that

$$B\left(a + \frac{r}{2} \cdot v, \frac{r}{2} - 2\varepsilon\right) \cap H = \emptyset. \tag{3.5}$$

Then H is $r/2$-ball porous at a, a contradiction.

To prove (3.5), suppose on the contrary that there exists $x \in B(a + r/2 \cdot v, r/2 - 2\varepsilon) \cap H$. By the choice of J, there exists $x^* \in J$ such that $\|x - x^*\| < \|x - a\|^2$. Denote $b := a + 2\varepsilon v$ and distinguish two cases.

If $\|x - b\| < 2\varepsilon$, then $\|x - a\| < 4\varepsilon$ and therefore $\|x - x^*\| < 16\varepsilon^2 < \varepsilon$ (since $\varepsilon < r/4 < 1/40$). Consequently, $x^* \in B(a + r/2 \cdot v, r/2 - \varepsilon) \subset B(a + rv, r - \varepsilon)$, a contradiction.

If $\|x - b\| \ge 2\varepsilon$, then $\|x - a\| \le 2\varepsilon + \|x - b\| \le 2\|x - b\|$ and thus $\|x - x^*\| \le 4\|x - b\|^2$. Put $\rho := r - 4\varepsilon$. Since $x \in B(b + \rho/2 \cdot v, \rho/2) = B(a + r/2 \cdot v, r/2 - 2\varepsilon)$, fact (F) implies that

$$x^* \in B(b+\rho v,\rho) = B(a+(r-2\varepsilon)v, r-4\varepsilon) \subset B(a+rv, r-\varepsilon), \tag{3.6}$$

a contradiction. □

Corollary 3.11. *Let X be a Hilbert space and let $S \subset X$ be a Suslin set which is not ball small. Then there exists a closed set $F \subset S$ which is not ball small.*

Finally, note that Theorem 2.3 can be easily applied also to the system of σ-cone porous sets in an arbitrary Banach space (by a cone porous set, we mean a set which is α-cone porous for some $\alpha > 0$; see [5] for the definition and [1] for some properties of α-cone porous sets in Hilbert spaces). On the other hand, it seems that Theorem 2.3 can be applied neither to the (more interesting) related system of cone small sets (cf. [6]) nor to the system of σ-cone supported sets in nonseparable Banach spaces.

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However, more and more researchers have found that the financial environment is not ruled by mathematical distributions or statistical models. In such situations, some attempts have also been made to develop financial engineering models using intelligent computing approaches. For example, an artificial neural network (ANN) is a nonparametric estimation technique which does not make any distributional assumptions regarding the underlying asset. Instead, ANN approach develops a model using sets of unknown parameters and lets the optimization routine seek the best fitting parameters to obtain the desired results. The main aim of this special issue is not to merely illustrate the superior performance of a new intelligent computational method, but also to demonstrate how it can be used effectively in a financial engineering environment to improve and facilitate financial decision making. In this sense, the submissions should especially address how the results of estimated computational models (e.g., ANN, support vector machines, evolutionary algorithm, and fuzzy models) can be used to develop intelligent, easy-to-use, and/or comprehensible computational systems (e.g., decision support systems, agent-based system, and web-based systems)

This special issue will include (but not be limited to) the following topics:

- **Computational methods**: artificial intelligence, neural networks, evolutionary algorithms, fuzzy inference, hybrid learning, ensemble learning, cooperative learning, multiagent learning
- **Application fields**: asset valuation and prediction, asset allocation and portfolio selection, bankruptcy prediction, fraud detection, credit risk management
- **Implementation aspects**: decision support systems, expert systems, information systems, intelligent agents, web service, monitoring, deployment, implementation

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