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Isometric classification of norms in rearrangement-invariant function spaces

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Abstract: Suppose that a real nonatomic function space on $[0, 1]$ is equipped with two rearrangement-invariant norms $\|\cdot\|$ and $\|\|\cdot\|\|$. We study the question whether or not the fact that $(X, \|\cdot\|)$ is isometric to $(X, \|\|\cdot\|\|)$ implies that $\|f\| = \|\|f\|\|$ for all f in X . We show that in strictly monotone Orlicz and Lorentz spaces this is equivalent to asking whether or not the norms are defined by equal Orlicz functions, resp. Lorentz weights.

We show that the above implication holds true in most rearrangement-invariant spaces, but we also identify a class of Orlicz spaces where it fails. We provide a complete description of Orlicz functions $\varphi \neq \psi$ with the property that L_φ and L_ψ are isometric.

Keywords: isometries, rearrangement-invariant function spaces, Orlicz spaces, Lorentz spaces

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