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On τ -extending modules

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Abstract: In this paper we introduce the concept of τ -extending modules by τ -rational submodules and study some properties of such modules. It is shown that the set of all τ -rational left ideals of ${}_R R$ is a Gabriel filter. An R -module M is called τ -extending if every submodule of M is τ -rational in a direct summand of M . It is proved that M is τ -extending if and only if $M = \text{Rej}_M E(R/\tau(R)) \oplus N$, such that N is a τ -extending submodule of M . An example is given to show that the direct sum of τ -extending modules need not be τ -extending.

Keywords: torsion theory; τ -rational submodules; τ -closed submodules; τ -extending modules

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