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Limit points of arithmetic means of sequences in Banach spaces

Comment.Math.Univ.Carolinae 41,1 (2000) 97-106.

Abstract: We shall prove the following statements: Given a sequence $\{a_n\}_{n=1}^{\infty}$ in a Banach space $\mathbf{X}$ enjoying the weak Banach-Saks property, there is a subsequence (or a permutation) $\{b_n\}_{n=1}^{\infty}$ of the sequence $\{a_n\}_{n=1}^{\infty}$ such that

$$\lim_{n\to\infty}\frac{1}{n}\sum_{j=1}^{n} b_j = a$$

whenever a belongs to the closed convex hull of the set of weak limit points of $\{a_n\}_{n=1}^{\infty}$. In case $\mathbf{X}$ has the Banach-Saks property and $\{a_n\}_{n=1}^{\infty}$ is bounded the converse assertion holds too. A characterization of reflexive spaces in terms of limit points and cores of bounded sequences is also given. The motivation for the problems investigated goes back to Lévy laplacian from potential theory in Hilbert spaces.

Keywords: Banach-Saks property, arithmetic means, limit points, subsequences, permutations of sequences
AMS Subject Classification: 46B20, 40H05, 40G05, 47F05