A Fundamental Geometry of Quantum Physics

Daniel C. Galehouse

Dedicated to Prof.Dr. Constantin UDRIŞTE on the occasion of his sixtieth birthday

Abstract

Quantum mechanics is well established as a mathematical description of the properties and interactions of matter. A complete understanding has been elusive and unsatisfactory. The object of this talk is to consider the intellectual hypothesis of a fundamental underlying geometrical system. The direct association of quantum mechanics with geometry leads to a number of new concepts and results. Classical mechanics is dropped and quantization is introduced intrinsically from the beginning. The conceptual development follows systematically from the association of geometry with quantum motion. A modification to the common electromagnetic gauge simplifies the quantum theory. The phase of the wave function is transfered to the vector potential. Subsequently, the magnitude of the wave function is transfered to a suitable conformal factor. The invariant equations of the conformal parameter are applied to the quantum geometrical structure. This leads to a demonstration that the probability density is proportional to the square of the wave function. Having understood this, the four dimensional theory is dropped and a transition is made to the more robust five dimensional theory. Standard conformal effects in electrodynamics are noted. The proper time is introduced as a fifth dimension and a quantum field equation is uncovered. A natural set of geodesics is displayed and these are used to describe the motion. Equivalence is satisfied for multiple fields. It is shown that certain source effects, developed from the conformal structure of the system, are intrinsic. The geometrically allowed interactions have the expected structure of electromagnetism and gravity. A particular simple case is worked out to get quantum source terms. The resulting field equations are covariant, quantized descriptions of interacting particles. A number of interesting properties are brought out and applications to astrophysics, cosmology and interaction theory are expected.

Mathematics Subject Classification: 81T10
Key words: applied Riemannian geometry, unified fields

1 Introduction

Integration of quantum phenomenology with electrodynamics, and gravitation has been persistently difficult. Studies have been made of several geometrical theories of

Balkan Journal of Geometry and Its Applications, Vol.5, No.1, 2000, pp. 93-106

quantum mechanics with the object of understanding if such a geometrical theory could allow the integration of gravitational physics. Indications are that inconsistencies in the use of classical mechanics preclude the development of more complex theories. To avoid the difficulties, it has been found necessary to reject any formal transition from classical mechanics to quantum mechanics. The quantization method, while it may be expedient in flat space, cannot be made to work with gravitational forces. These problems motivate this talk. The object is to use direct methods to associate a geometrical theory with gravitational, electromagnetic and quantum observations. [1]

To pursue a theoretical development without using a classical basis, any tests to confirm or falsify must be based directly on experiment without the intercession of standard phenomenology. At present, all known accepted experiments, if carried out to sufficient precision, confirm the quantum paradigm over any classical fundamental theory. Consequently, it is sensible to suppose that any construction based on classical arguments should be doubted unless it is supported by a quantum explanation or observation. This mental dictum is not undertaken lightly as the implied changes in thought pattern are radical. Moreover, the effects are integrated throughout the fabric of physics. In the meanwhile, the understanding of quantum mechanics is held hostage. While Feynman would probably comment that nobody really understands quantum theory, the real problem is that God doesn't know anything about classical mechanics.

For an independent derivation, some element of fundamental quantum mechanics must be associated with a well chosen element of geometry. The natural covariance and transformations of the geometry should do the work to create a relativistic quantum theory. Standard classical differential geometry seems to be sufficient and will be used exclusively in this talk. A careful interpretation of quantum experiments is absolutely necessary and a certain amount of improved understanding is implied. The geometry integrates well and contributes to the solution of the problem of quantum incomprehensibility.

Experimental tests of equivalence are convincing. The exactness of these tests for different forms of binding energy supports a fundamental geometrical mechanism. While the present discussion is limited to the gravitational and electromagnetic interactions, the predictions are strong enough to see essential quantum effects play out. The results are in agreement with experiment, given the limitations imposed by the assumed scope. The structure is fully quantized for both electromagnetism and gravity. Extensions to spin, weak and strong interactions are left for later work.

2 A basic assumption

Searching systematically through the myriad of geometrical systems is beyond the scope of this talk. It is expedient to begin with an elementary assumption and develop useful results. The integral curves of a probability four vector are taken as a sense of motion. It is necessary to find suitable geodesics in a usable geometry. Without interaction or interference, a quantum plane wave in a flat space has everywhere

[1] References and background discussion are available in the preprints quant-ph@xxx.lanl.gov /9412012, gr-qc@xxx.lanl.gov /9512034, and gr-qc@xxx.lanl.gov /9512035.

parallel flow lines which match the inertial geodesic field of the underlying space. Following this, transformations of the space can be used to develop more complex physics. To the general coordinate transformations used for relativity are added the conformal transformations. These seem to be enough for the quantum theory.

Inertial motion

Forced motion

In this sense, the whole wave function is mapped to an entire geodesic congruence. This construction avoids most of the pitfalls of the neo-quantum theories. In particular, the question of hidden variables is avoided because the entire congruence is the basis of interaction. No individual geodesic can be physically selected in preference to others and no single trajectory can be evaluated as to the presence of a particle. A semi-classical radiation theory is precluded because the electromagnetic interaction is attached to the congruence as a whole and not to a collection of trajectories with assigned probabilities. Each particle fills the universe. A collection of such particles are countable in the sense of being discreet in number but not in the sense that they may be said to exist at any point in space time. Localization is imperfect and dynamical. A localized probability distribution can still be used for quantum observations.

3 The essential electromagnetic gauge transformation

The standard form of the quantum wave equation is not conducive to geometrical theories. In particular, there is no known direct geometrical interpretation for the wave phase. The conventional gauge transformation can be used to produce a better form. For simplicity it is best to start with the Klein-Gordon equation

$$(i\hbar\frac{\partial}{\partial x_\mu} - eA^\mu)(i\hbar\frac{\partial}{\partial x^\mu} - eA_\mu)\psi - m^2\psi = 0,$$

which has the current

$$P^\mu = \frac{1}{2m}\psi^*(i\hbar\frac{\partial}{\partial x_\mu} - eA^\mu)\psi - \frac{1}{2m}\psi(i\hbar\frac{\partial}{\partial x_\mu} + eA^\mu)\psi^*.$$

The gauge transformation is

$$\psi = e^{i\phi}\psi', A_\mu = A'_\mu - \hbar\frac{\partial\phi}{\partial x^\mu}$$

and the parameter ϕ can be chosen so that ψ' is real. This process introduces the entire phase into A'_μ. The resulting equations for ψ' and A'_μ as a fixed gauge quantity are

$$m^2 - e^2A'^2 = \hbar^2[(\ln\psi')_{,\mu}(\ln\psi')_{,}{}^{\mu} + (ln\psi')_{,}{}^{\mu}{}_{\mu}]$$

$$P^\mu = -\frac{e}{m}A'^\mu\psi'^2.$$

The simplification to the form of the current is essential. Both equations are fully real. In this gauge, quantum motion is everywhere parallel to the vector potential. The probability curves can match congruence induced motion. The standard form of the Klein-Gordon equation can be reconstructed by reversal of the transformation. In this way, the phase is associated with the intrinsic motion and is separate from the amplitude.

4 The essential conformal effect on density

The diffraction of a finite mass particle will cause variations in the probability density. A local observer counts particles using his own sense of scale size. It is now to be shown that geometrical consistency requires the wave function amplitude, $|\psi|$, to be coupled to a conformal parameter.

Consider a simple diffraction experiment consisting of a particle that leaves a source, traverses a target and arrives on a screen. There will be an observed density found by counting arrivals in a region on the target. The flux to the target is measured using a standard laboratory metric $\dot{g}_{\mu\nu}$ and results in a numeric measure of particle density.

Diffraction experiment

Suppose that this metric is modified by a conformal factor λ giving an effective metric $\lambda\dot{g}_{\mu\nu}$. The unit scale size is changed for the new observer and the resulting observed density depends on λ. At a point of the screen, each of the local coordinates x, y, z, t is rescaled by $\sqrt{\lambda}$. Assuming motion in the z direction, the velocity is unchanged.

$$\frac{\Delta z\sqrt{\lambda}}{\Delta t\sqrt{\lambda}} = \frac{\Delta z}{\Delta t} = v.$$

The transverse directions are each scaled by $\sqrt{\lambda}$ modifying the observed density by λ.

Gauge scaling

An increase in the numerical magnitude of the observer's metric reduces the physical size of a unit surface element. The remeasured surface density of particles is reduced from $|\psi^2|$ to $|\psi|^2/\lambda$. This implies a gauge transformation between two descriptions of the same quantum wave function, $(|\psi|^2, \dot{g}_{\mu\nu}) \rightarrow (|\psi|^2/\lambda, \lambda\dot{g}_{\mu\nu})$. The simplifying gauge choice $\lambda = |\psi|^2$ reduces the probability density to a geometrical effect. Thus $(|\psi|^2, \dot{g}_{\mu\nu}) \rightarrow (1, \lambda\dot{g}_{\mu\nu})$ which can be represented as $(1, |\psi|^2\dot{g}_{\mu\nu})$, where $|\psi|^2$ is the wave function before the transformation.

In practice, the particle wave function amplitude is assigned a geometrical meaning by using a particle specific metric. The observer's gauge is left at the standard value. The probability variations become effects in the particle metric. The cost of the construction is is high because of the large number of metrics introduced, but the physical properties are well described by this construction. The equivalence of the electrodynamic, quantum and gravitational fields requires, at least at some level, an individual set of three field quantities for each particle. The multi-metric structure is essential for quantum effects and geometrical equivalence.

It is worth noting that the probability density must go as the square root of the conformal factor raised to the power of the signature of the metric,

$$P^{\mu} \sim (\sqrt{\lambda})^{(3-1)} = \lambda.$$

This is required for a covariant theory of particle density. To obtain the observed quantum result, it must also be that λ is equal to $|\psi|^2$, with ψ coming from a linear wave equation. The association of the geometrical quantity $\sqrt{\lambda}$ with a linear equation will complete the structure. In this way it is to be shown that, the probability density can only be proportional to the square of the wave function.

5 Conformal wave equations

If a physical wave function is coupled to the conformal factor, then characteristic equations for the conformal factor should be reflected in the properties of the quantum field equation. It is natural to look at the way that the conformal factor can appear in the Riemann tensor.

Consider a simplified metric of the form $\omega\eta_{\alpha\beta}$ where ω is a scalar function of the coordinates and $\eta_{\alpha\beta}$ is the standard local metric with ± 1 on the diagonal and 0 elsewhere. The scalar of curvature in n dimensions is

$$R = (n-1)\frac{1}{\omega^2}\frac{\partial^2 \omega}{\partial x^a \partial x_a} + \frac{(n-1)(n-6)}{4}\frac{1}{\omega^3}\frac{\partial \omega}{\partial x^a}\frac{\partial \omega}{\partial x_a}.$$

Let $\omega = \psi^p$ with p and ψ real. The substitution gives

$$R\psi^p = (n-1)p\left\{\frac{1}{\psi}\frac{\partial^2 \psi}{\partial x^a \partial x_a} + \left[\frac{n-2}{4}p - 1\right]\frac{1}{\psi^2}\frac{\partial \psi}{\partial x^a}\frac{\partial \psi}{\partial x_a}\right\}.$$

Choose $p = \dfrac{4}{n-2}$ to replace the nonlinear equation in ω by a linear equation in ψ.

$$R\psi^p = \frac{4(n-1)}{(n-2)\psi}\frac{\partial^2 \psi}{\partial x^a \partial x_a}.$$

There are two cases of interest.

Case #1, $n = 4, p = 2, R\psi^2$ constant:

$$\psi(R\psi^2) = 6\frac{\partial^2 \psi}{\partial x^a \partial x_a}.$$

Set

$$R\psi^2 = -6m^2$$

to get

$$\frac{\partial^2 \psi}{\partial x^a \partial x_a} + m^2\psi = 0.$$

This equation is identified with the Klein Gordon equation in four space. The construction is only schematic as the electromagnetic quantities must be added. Since there is no natural Riemannian structure in four space for the electromagnetic field, these extra effects must appear as non-Riemannian terms. A type of quantum Weyl theory is relevant and the Klein-Gordon equation is then a natural extension of the above characteristic equation. A constant universal particle mass can only be introduced when the four space metric satisfies a consistency condition relative to the natural quantum scale size. Properly executed, this structure reproduces quantum behavior. It is important as a four dimensional description of a mechanism that is certainly more complicated.

Case #2, $n = 5, p = 4/3, R = 0$:

$$\frac{\partial^2 \psi}{\partial x^a \partial x_a} = 0.$$

Set

$$\frac{\partial \psi}{\partial x^5} = im\psi$$

and get

$$\frac{\partial^2 \psi}{\partial x^\alpha \partial x_\alpha} + m^2 \psi = 0.$$

This is again a linear wave equation. The important difference is the way that m is introduced. Here it is associated with a natural scale connected to the fifth dimension. As a parameter of five space, it insures that scale sizes of quantum particles (i.e. Compton wavelengths) can be universally and consistently defined. The same fifth coordinate is chosen for all particles. This five dimensional description is taken as the fundamental structure.

The expansion of the above, for a standard five metric (instead of η_{ab}), automatically introduces the vector potential, A_μ, correctly and with proper ordering. The Klein-Gordon equation results without special construction of any kind. In addition, very small, corrections to the effective mass appear. These corrections seem to allow the particle congruence to have a space-like motion for distances of order the Planck length. Because of the small scale size, direct laboratory observation seems unlikely.

6 Conformal transformations of interacting fields

Before attempting to understand interaction mechanisms, it is instructive to note some of the common effects of a conformal transformation. Electrostatic solutions are generated by the method of images, the Schwartz-Christoffel transformation and other elements of complex function maps. These studies come to completion with the conformal invariance of Maxwell's equations. While invariance applies to the homogeneous field equations, the source densities of the inhomogeneous equations must change. In fact, a conformal transformation can take a solution without local charges into one with local charges. The inversion of a uniform electric field produces the field of a dipole. At some point it is natural to expect conformal effects to couple to the electric source densities and in the quantum case to the source wave functions.

Conformal effects are carefully eliminated from classical relativity. While the conformal invariance of the metric is asserted on physical grounds, real calculations are executed in an observers frame with fixed scale size. A calculated electromagnetic energy density must depend on the conformal factor. This interplay of conformal effects is implicit in the construction of quantum source terms. It turns out that the various physical effects can be synthesized effectively in five dimensions.

7 Why five dimensions?

A complete theory requires that a quantum field equation be assembled with both a source equation and a motion equation. Different pieces of the puzzle are found in different places. The Einstein-Maxwell equations can be derived from the consideration of nullity of the Riemmann tensor formed from the standard five metric. These are usually written

$$\Theta^{\kappa\lambda} = R^{\kappa\lambda} + \frac{1}{2}F^{\kappa\rho}F_{\rho}^{\cdot\lambda}$$

$$\Theta_5^{\cdot\kappa} = \frac{1}{2}F^{\kappa\lambda}|_\lambda,$$

where $\Theta^{\kappa\lambda} = 0$ and $\Theta_5^{\cdot\kappa} = 0$ for Einstein's derivation but $\Theta^{\kappa\lambda} = 8\pi\kappa\rho v^\kappa v^\lambda$ and $\Theta_5^{\cdot\kappa} = 2\pi e\rho v^\kappa$ for Kaluza's hypothesis.

It is intended that the five conformal waves and the resulting Klein-Gordon equation should complement the interaction mechanism and perhaps augment the source structure. The basic quantum assumption requires a geodetic motion that can project to the quantum probability current. It turns out that the classical five dimensional geodesics can be taken over directly.

8 Quantum five geodesics

A suitable form of geometrical motion has long been available from the classical five dimensional theory. The usual metric in the special gauge used here,

$$\gamma_{mn} = \begin{pmatrix} \dot{g}_{\mu\nu} - A_\mu A_\nu & A_\mu \\ A_\nu & -1 \end{pmatrix}$$

has a preferred geodesic that is particularly simple:

$$\frac{dx^\mu}{ds} = \dot{g}^{\mu\nu}A_\nu.$$

This is taken as the characteristic quantum motion and is combined with the conformal waves and the Einstein-Maxwell construction.

This motion, when projected onto the four space by ignoring the fifth coordinate, is invariant under three types of gauge transformation. An independent parameterization of this transformation can be taken as the three functions ω, χ and λ. The first, ω multiplies the full five-metric, the second, χ multiplies the vector potential and λ remains as a gauge factor for the four-metric. After a transformation by ω, χ, and λ, the standard five-metric becomes

$$\gamma_{mn}(\omega, \lambda, \chi) = \omega \begin{pmatrix} \lambda\dot{g}_{\mu\nu} - \chi A_\mu \chi A_\nu & \chi A_\mu \\ \chi A_\nu & -1 \end{pmatrix}$$

but has geodesics with indistinguishable projection: $\frac{dx^\mu}{ds} = \dot{g}^{\mu\nu}A_\nu\frac{\chi}{\lambda}$. The indeterminate scale factor can always be absorbed into the path parameterization.

The identifiable forces that are inferred from the observed trajectory when projected into four space can be found from the total derivative. This must be done after normalizing the path parameter. Let the new parameter, w, be scaled so that $dw^2 = \dot{g}_{\mu\nu}dx^\mu dx^\nu$, then

$$\frac{du^\mu}{dw} + \dot{\left\{ \begin{matrix} \mu \\ \epsilon\lambda \end{matrix} \right\}} u^\epsilon u^\lambda = \frac{e}{m} F^{\mu\lambda} u_\lambda + \frac{e}{m}(\xi^\lambda A^\mu - \xi^\mu A^\lambda) u_\lambda + \frac{e}{m}(\xi - 1) F^{\mu\lambda} u_\lambda .$$

Here

$$u^\mu = \frac{dx^\mu}{dw},$$

$$\dot{\left\{ \begin{matrix} \mu \\ \epsilon\lambda \end{matrix} \right\}} = \dot{g}^{\mu\rho} \frac{1}{2} \left(\frac{\partial \dot{g}_{\epsilon\rho}}{\partial x^\lambda} + \frac{\partial \dot{g}_{\rho\lambda}}{\partial x^\epsilon} - \frac{\partial \dot{g}_{\epsilon\lambda}}{\partial x^\rho} \right),$$

$$F_{\lambda\beta} = \frac{\partial A_\lambda}{\partial x^\beta} - \frac{\partial A_\beta}{\partial x^\lambda},$$

and

$$\xi = \frac{m}{|A|e}.$$

This proves directly the idea of extended equivalence. Factors of ω, χ, or λ will change the relative sizes of the above terms which represent gravitational, electromagnetic or quantum effects. The interplay of these forces, assuming a fixed trajectory, gives a concept of equivalence for quantum particles. An apparent deflection can come from any field depending on the orientation of the observer's frame and the relative conformal factor.

In functionality the gauge factors ω, χ, or λ must be related to the geometrical origination of the field sources. Perhaps there may be a way to parameterize the dependence of these quantities on the currents that generate the interaction. Because these factors are four scalars, some form of covariant interaction is assured. Since the free fields obey the Einstein-Maxwell equation, the standard inverse square solutions are expected away from the sources. The identification of electromagnetism and gravity is hard to avoid. The question remaining is whether the source structure itself is acceptable. A complete analysis of the effects of the conformal factors can be carried out by expansion of ω, χ, and λ to second order. The considerations are beyond the scope of this talk. A simplification is presented which shows that the essential aspects of the interactions are indeed present.

9 Structure of the field equations

A direct assembly is now possible. First, use the five dimensional scalar waves and let them correspond to the quantum mechanical Klein-Gordon equation. Choose $\chi = 1$, $\lambda = 1$, $R = 0$, $n = 5$, $\omega = (\psi e^{im\tau})^{4/3}$. One gets directly,

$$\frac{1}{\sqrt{-\dot{g}}}(i\hbar \frac{\partial}{\partial x^\mu} - eA_\mu)\sqrt{-\dot{g}} g^{\mu\nu}(i\hbar \frac{\partial}{\partial x^\nu} - eA_\nu)\psi = [m^2 + \frac{3}{16}(\dot{R} - \frac{e^2}{m^2} F_{\alpha\beta}F^{\alpha\beta})]\psi.$$

The term additive to the mass is very small but may be observable in very high field regions. Otherwise, the equation is identical with the usual quantum equation. The additional conformal factors can be used to describe the force. The simplest continuation is to assume that there is a true five metric, related to the classical five metric by transformation with ω, χ, and λ. A simple theory results if this true five metric is taken curvature free. It is simple and expedient to assume that the conformally transformed five space is flat.

$$\Theta^{mn}\left[\omega\begin{pmatrix}\lambda\dot{g}_{\mu\nu}-\chi A_\mu\chi A_\nu & \chi A_\mu\\ \chi A_\nu & -1\end{pmatrix}\right]=0.$$

This five-Ricci tensor is set to zero. This equation in turn implies a non-zero curvature for the standard five metric. The value can be found by expanding the above condition and rearranging terms. A long calculation shows that it can be written in the form

$$\Theta^{mn}\left[\begin{pmatrix}\dot{g}_{\mu\nu}-A_\mu A_\nu & A_\mu\\ A_\nu & -1\end{pmatrix}\right]=T^{mn}(\omega,\chi,\lambda),$$

where dependency of ω, χ, λ on the external source structure is implied.

10 Field equations first guess

In the above calculation, the dependence on χ and λ produce terms with unwanted side effects. Factors that depend on the test particle motion rather than on the source particle motion appear in the source equation and are difficult to cancel. It may be that these are part of some sort of geometrical renormalization. Since they do not generate straightforward source effects they are dropped here. The term in ω remains. Since the curvature operator is second order, expansion to second order is sufficient. The zero order term has no effect, and the first order term does not produce a useful result beyond that of the second. It is suitable to choose $\omega = B_{nm}x^m x^n$, $\lambda =$ constant, $\chi =$ constant. The available tensors that might be used to make up B_{nm} are either the particle five metric or the metric of the standard neutral observer. A somewhat contrived but revealing assumption is

$$B_{ij}=\frac{|\psi|^2 f}{2-\frac{e^2}{m^2}A^2}\left[\begin{pmatrix}\dot{g}_{\mu\nu} & \\ & -1\end{pmatrix}-\frac{7}{8}\begin{pmatrix}\dot{g}_{\mu\nu}-A_\mu A_\nu & A_\mu\\ A_\nu & -1\end{pmatrix}\right],$$

where f, χ, and λ are numerical parameters that can be adjusted to fit known interaction constants. Direct calculation gives

$$R^{\alpha\beta}=8\pi\kappa\left[F^{\alpha}_{\cdot\mu}F^{\mu\beta}+m|\psi|^2\frac{e^2}{m^2}A^\alpha A^\beta+m|\psi|^2\frac{\left(1-\frac{e^2}{m^2}A^2\right)}{\left(2-\frac{e^2}{m^2}A^2\right)}\dot{g}^{\alpha\beta}\right]$$

and

$$F^{\alpha\mu}|_\mu=4\pi e|\psi|^2A^\alpha.$$

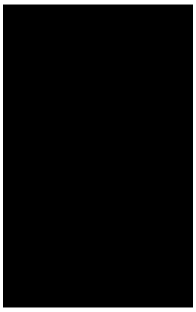
The equation in $R^{\alpha\beta}$ is the Einstein field equation written for the Ricci tensor. The first source term on the right is scaled to the accepted value of the gravitational strength of the electromagnetic energy density. The second source term is apparently the quantum limit of the incoherent matter. The changes involve the use of the quantum density $|\psi|^2$ instead of the incoherent particle density and the velocity scaled vector potential $\frac{e}{m}A^\alpha$ rather that the classical kinematic velocity. The last term is of the form of a cosmological constant except that it is source dependent. Because of the factor $1 - \frac{e^2}{m^2}A^2$ it is zero for motion in the classical limit. This new term cannot be produced by the direct procedural quantization of any classical theory. It is interesting because it can be negative, depending on the exact form of the quantum state. Any repulsive gravitational effect, even as small as this one could be important. A comparison with experiment is in order. The expected effects appear to be too small for earth based tests but should be important for cosmology or astrophysics.

The equation in $F^{\alpha\beta}$ is exactly Maxwell's inhomogeneous equation except that the incoherent source term is replaced by the quantum probability current. This is in agreement with the basic tenants of quantum electrodynamics. The interaction constant is adjusted to agree with the observed value of the electric charge. While there is apparently no need for mass renormalization in this theory, the issue of effective charge renormalization due to multi-particle interactions is not addressed. The displayed equation is sufficient for this stage of development. All together, these represent a geometrically unified source system for gravitational and electrodynamic interactions of quantum particles.

11 Applications

While corrections to the standard theory are small, certain effects can be expected especially where large mass concentrations can offset the small value of Planck's constant. The first question is whether the repulsive gravitational term mimics an effective cosmological constant. Is it of the correct character and strength to resolve the current controversy over the openness of the universe and the apparent continuing acceleration of expansion? Such observations may be evidence for the fifth dimension or for the flatness that could accompany it. In black hole physics, the classical concept of an absolute horizon cannot be maintained. The conjectured flatness of five space indicates the absence of any real singularity. Information may be conserved and displacement of matter, both in and out of a super-massive object is suggested. In particular, a repulsive effect, especially near large rapidly moving objects, may be significant. An immediate expectation is to reduce the overall gravitational force and cause the horizon to dip down a bit into the central area. Moreover since the underlying matter must have a very high temperature and ultra-relativistic velocity, even a small displacement below the material surface should cause outward particle motion.

The pattern of ejection must be supposed to depend on the state of the object, its size, rotation rate, and mass. For non-rotating objects, the effects of thermodynamic fluctuation or quantum noise could lead to a local inward displacement of the horizon. Subsequent outward ejection may result in a spherical external object such as a globular cluster. Rotating objects can be imagined to have either relatively oblate or prolate horizons. These could lead to axial ejections as an astrophysical jet or to radial ejections, perhaps as a spiral galaxy.



Any of these may be modified by the correction term in the Klein-Gordon equation. These adjustments to the effective particle mass apparently correspond to the gravitational or electrodynamic production of particle pairs. No experimental information seems to be available. Particles may appear as some sort of tunneling process. The coupling between the wormhole like behavior of pair creation and the extreme gravitational effects at high mass are not yet understood.

From the theoretical side, this type of structure may be even more interesting for fundamental interaction theory. The electromagnetic constant that appears in the geometrical derivation of the Maxwell equation relates two geometrical quantities. A calculation of its value may be possible and further understanding of its origin is likely. The flatness of single particle motion in five space is at least an intriguing possibility. If played out, it will have far reaching implications for fundamental physics.

12 Conclusions

This study is still a work in progress. Some of the results and the properties of this system are worth listing. The theory is fully and explicitly four covariant. The first quantization is complete and intrinsic. Second quantization of particles corresponds to the use of multiple metrics while the photon quantization is obtained by way of the time symmetric formalism. The interactions are finite and can be brought into agreement with gravity and electromagnetism. The Einstein structure for gravity and the Maxwell structure for electrodynamics are intrinsic. All fields are fully integrated geometrically. The incoherent matter source terms of Kaluza are replaced by quantum coherent source densities. It is an exact theory without need of a perturbation structure for its formulation. The classical limit is acceptable. Applications to astrophysics and interaction theory are expected.

Aknowledgements. A version of this paper was presented at Ninth Midwest Geometry Conference, November 5-7, 1999, University of Missouri, Columbia, USA.

Affiliation: Department of Physics, University of Akron
Mailing address: 15764 Galehouse Rd., Doylestown, Ohio 44230 USA
E-mail: dcg@uakron.edu