PVM Implementations of 3D Reconstruction Algorithms

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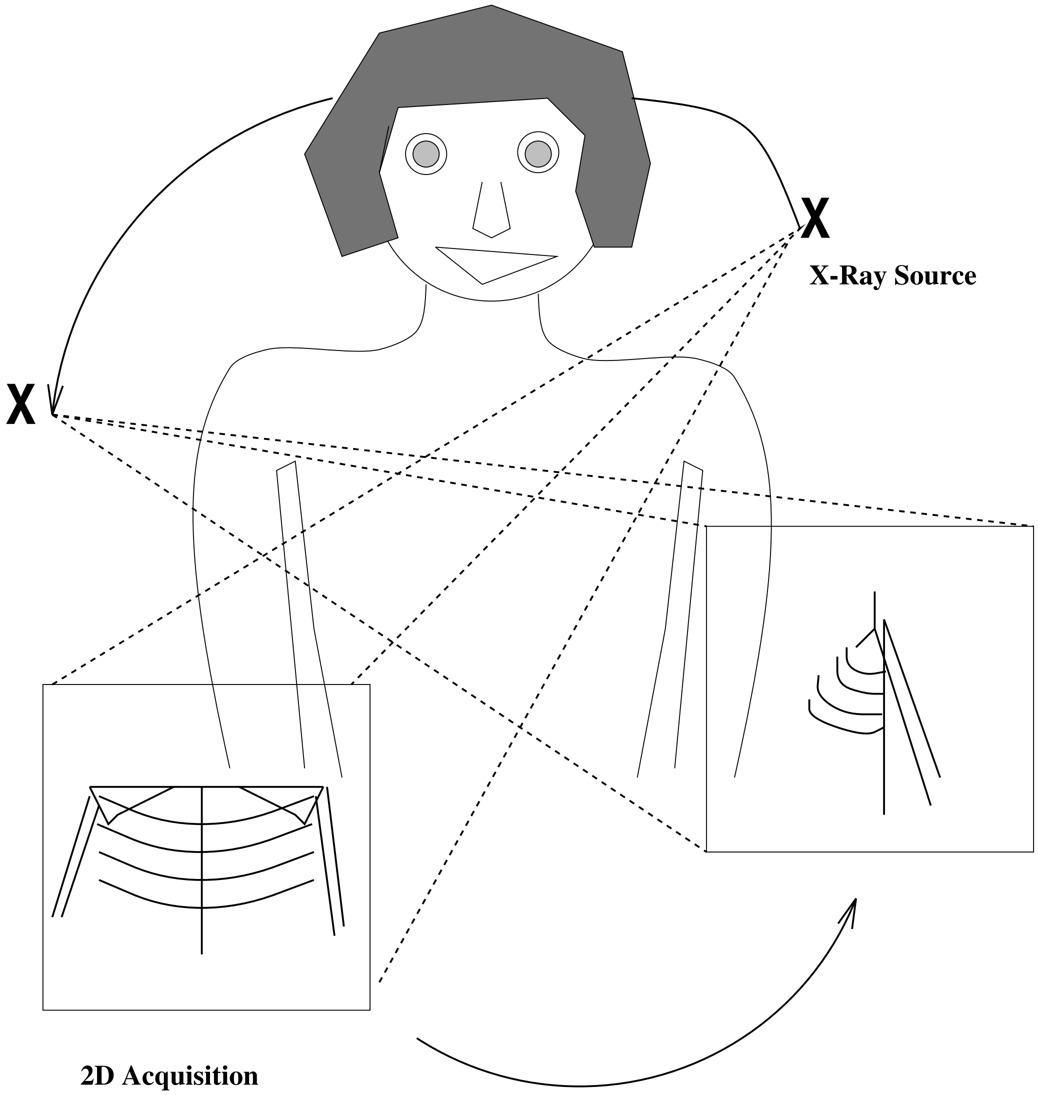
What is Cone-Beam X-Ray Tomography?



Aim : Vizualisation of a 3D Medical Images

Data: set of 2D Acquisitions

Principe: Reconstruction a 3D Image from the Data

Two acquisition protypes

- the DSR from the Mayo Clinic [Ritman80]
- the Morphometre from the GE-CGR and the LETI-CENG [SaintFelix92]

Problem of 3D Reconstrution Methods

- Divergent beam $\Longrightarrow$ Truly 3D Tomography
- Method:Truly 3D Tomography $\Longleftrightarrow$ a generalisation of 2D Tomography
- Choice of Method: Analytic, Algebraic and Statistical
- 3D Image Size: 536 Mbytes for a 512^3 Image
- Data Size: 268 Mbytes for 256 512^2 Acquisitions
- $\Longrightarrow$ a Big storage and a lot of Input-Output Exchange
- Computation times: for instance 10^4 seconds to reconstrute a 128^3 Image from 100 2D Acquisitions 128^2

A good solution $\Longrightarrow$ the Parallel Computing

State of art

- Transputer Network [Atkins93]
  3D PET (Positron Emission Tomograph)

- MIMD Computer
  - NCUBE [Zapata90]
    Analytic Algorithm
  - IPSC i860 [Charles93]
    Filtered Backprojection Algorithm
  - IPSC 2 [Chen91]
    Statistical Method for 3D PET
- CRAY X-MP Computer [Guerrini89]
  Filtered Backprojection Algorithm
- SIMD Computer: MasPar 16k [Miller93]
  Statistical Method for SPECT (Single Photon Emission Computed)

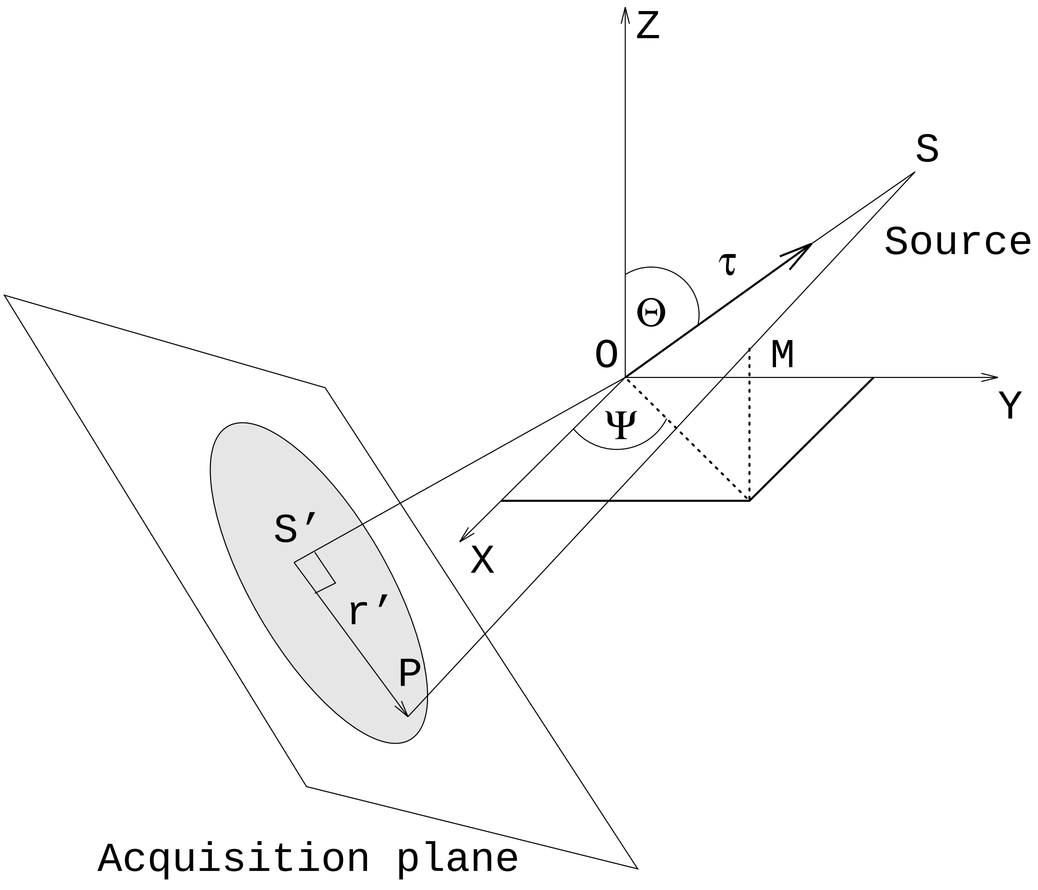
$\Longrightarrow$ Purpose of the Work

- Implementation of 3D Reconstruction Algorithms
- Using a Workstations Network
- Using of PVM with a topology on ring

Principe of 3D Tomography

Problem : Reconstruction a 3D Image from a set of 2D Acquisitions

2D Acquisition = projection of the 3D Image

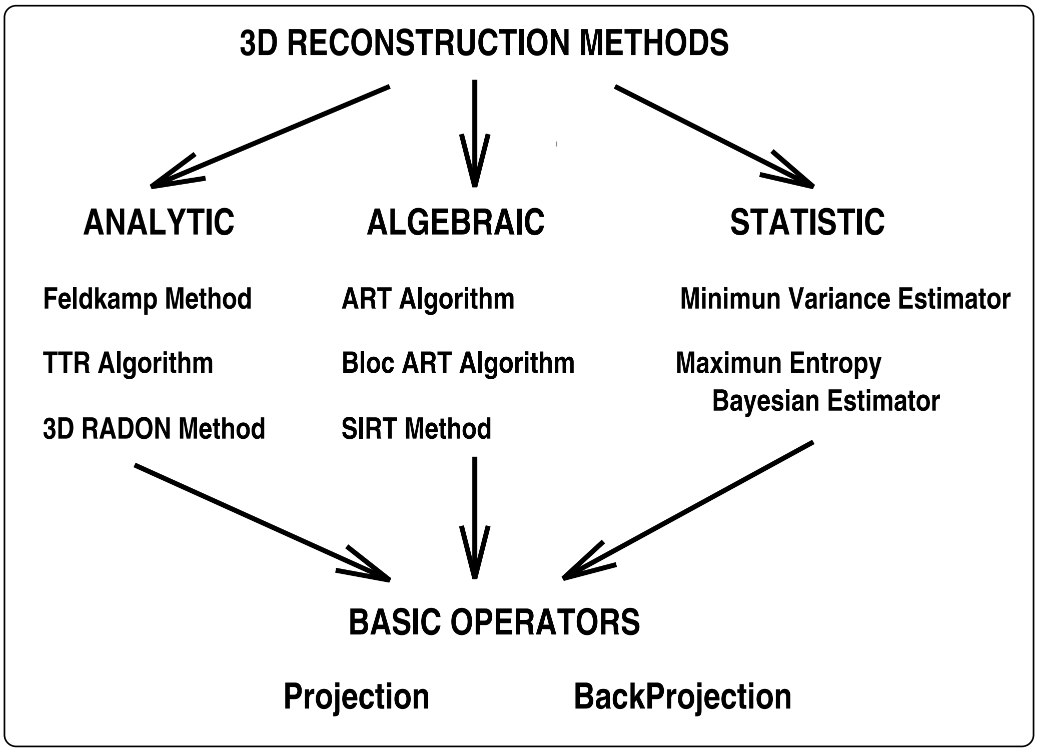


For each P(u, v): the projection value $P_\tau(u, v) = \int_{(SP)}$ 3D Object

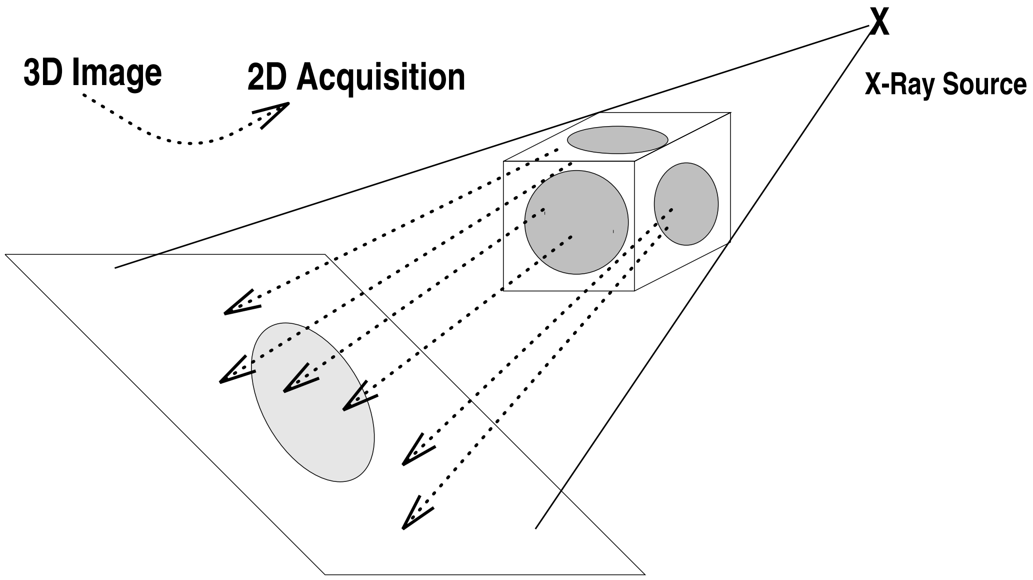
$\int_{(SP)} \Longleftrightarrow$ Integration on the straight line

Problem $\Longleftrightarrow$ Find a Function $f(x, y, z)$ from $\{P_\tau(u, v) \backslash (u, v) \in \Re^2, \tau \in \mathcal{T}\}$

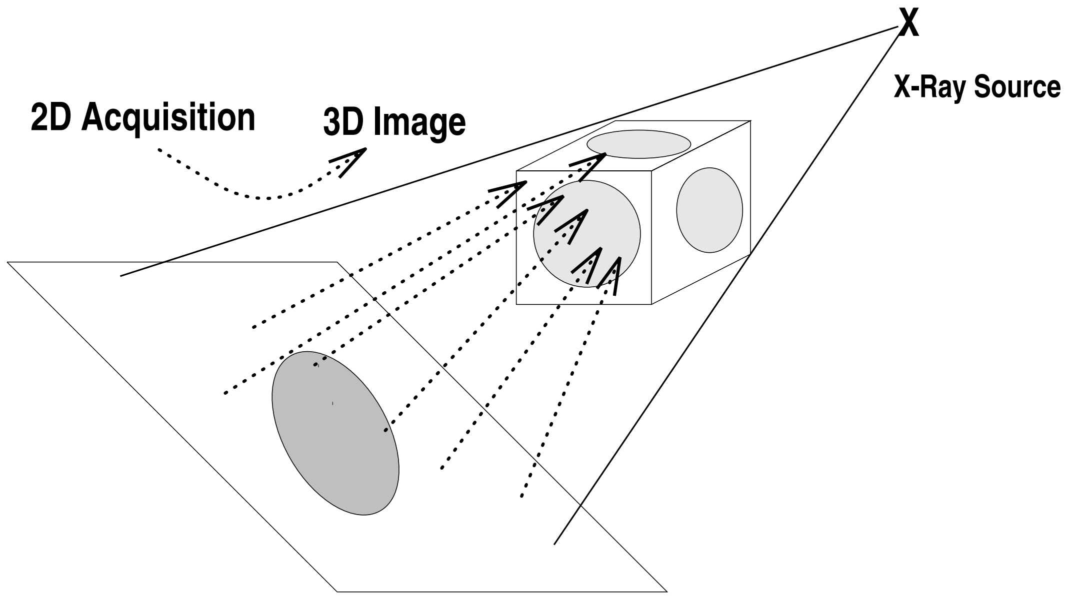
Reconstruction Methods



Basic Operators



(a) Projection

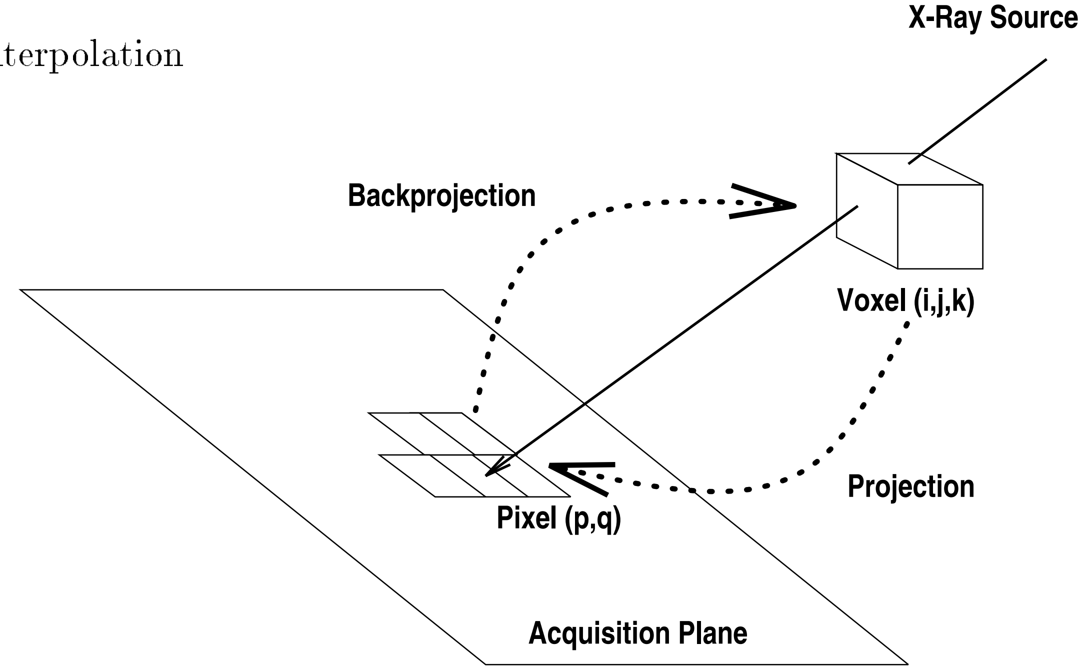


(b) Backprojection

Basic Operators

Using a "Voxel-Driven" Method with three steps:

- Adress calculation
- Ponderation
- Interpolation

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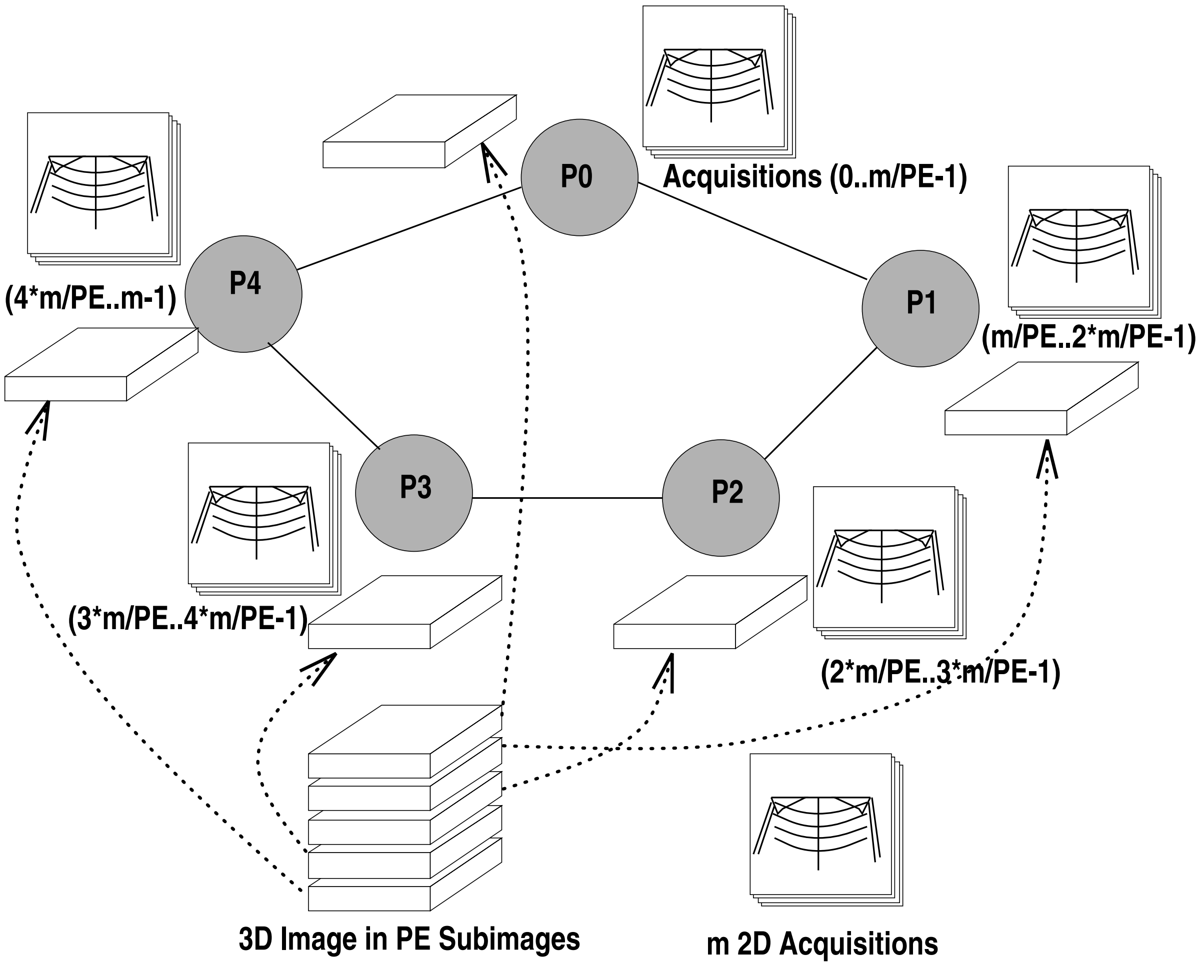
      Projection algorithm                    Backprojection algorithm

for(l=0; l< m;l++)                        for(l=0; l< m;l++)
  Create(Projection);                       read(Projection);
  for(k=0; k<N; k++)                        for(k=0; k<N; k++)
  for(j=0; j<N; j++)                        for(j=0; j<N; j++)
  for(i=0; i<N; i++)                        for(i=0; i<N; i++)
    calculate pixel(p,q) from voxel(i,j,k)    calculate pixel(p,q) from voxel(i,j,k)
    Projection[p+M*q]=Image3D[i+j*N+k*N*N];   Image3D[i+j*N+k*N*N]=Projection[p+M*q];
  write(Projection);                      write(Image3D);
```

Theoretical Cost of Sequential Implementations is: $O(m.N^3)$

Data Partition

Data: m 2D Acquisitions (M^2) and a 3D Image (N^3)
Target Machine: a Network of five SUN 4 Workstations



For PE Processors:

- m_{PE} Acquisitions of size M^2 ($m = m_{PE}.PE$),
- a Subimage of size $N_{PE}.N^2$ ($PE.N_{PE} = N$).

Communications Cost for a global Projection is:

- $(PE-1)\frac{m.M^2}{PE}$ if the 2D Acquisitions are communicated,
- $(PE-1)\frac{N^3}{PE}$ if the 3D Subimages are communicated.

The choice depends if $N^3 > m.M^2$.

Adaptative Data Partition for the Asynchronous Implementation

Initialization of Workstation Speed at S_0

After each Execution the Speed is S_i for the Workstation W_i

For the next Execution, we defined $S = \sum_{i=0}^{PE-1} S_i$

The Allocation of 3D Image is : $N^3 \frac{S_i}{S}$ for the the Workstation W_i

Theoretical Costs

Synchronous Implementation

- $O(N_{PE}.N^2.m)$ for the calculations,
- $O((PE-1)\frac{m.M^2}{PE})$ for the communication of the 2D projections.

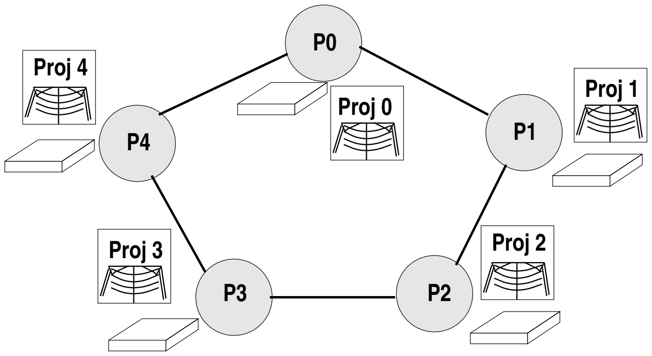
Asynchronous Implementation

- $O(N_{PE}.N^2.m)$ for the calculations,
- $O(\varepsilon.(PE-1)\frac{m.M^2}{PE})$ for the communication of the 2D projections.

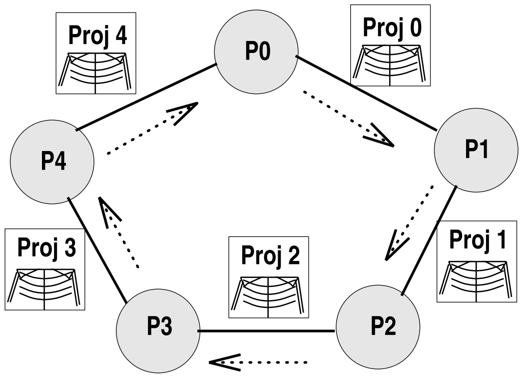
Adaptative Data Partition for the Asynchronous Implementation

- $O(N_{speed}.N^2.m)$ for the calculations,
- $O(\varepsilon.(PE-1)\frac{m.M^2}{PE})$ for the communication of the 2D projections.

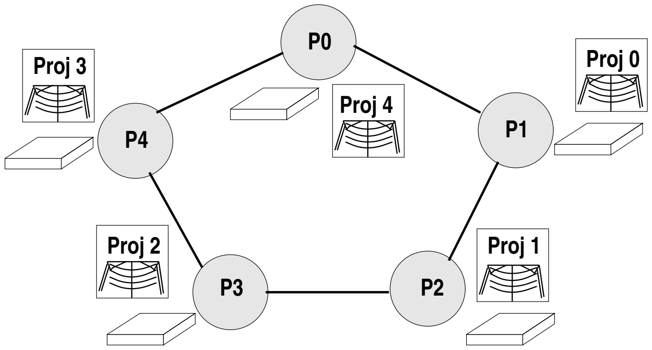
Synchronous Implementation



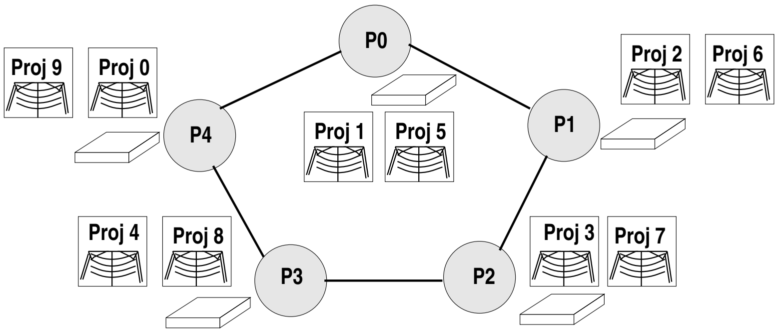
(c) Creation and local calculation step



(d) Communication step

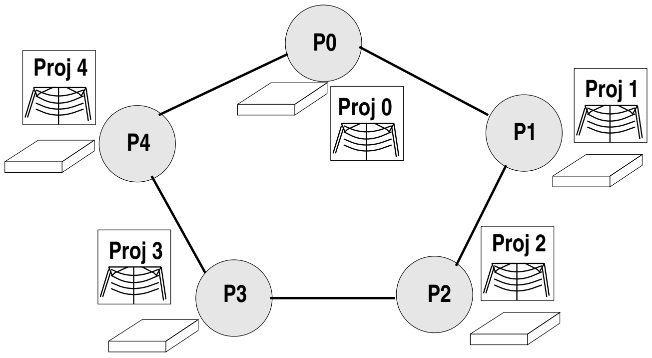


(e) Local calculation step

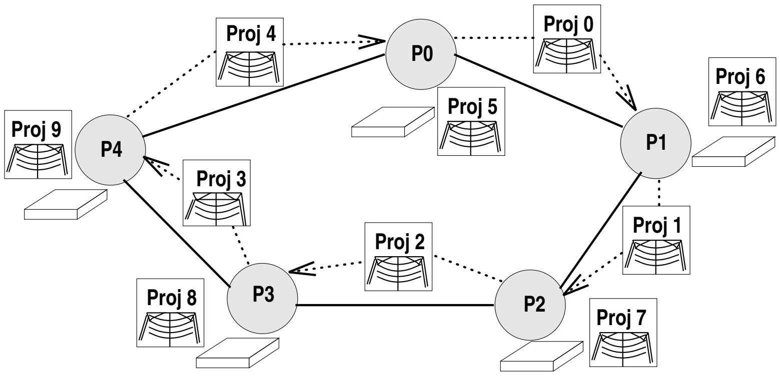


(f) Local calculation, writing and creation step

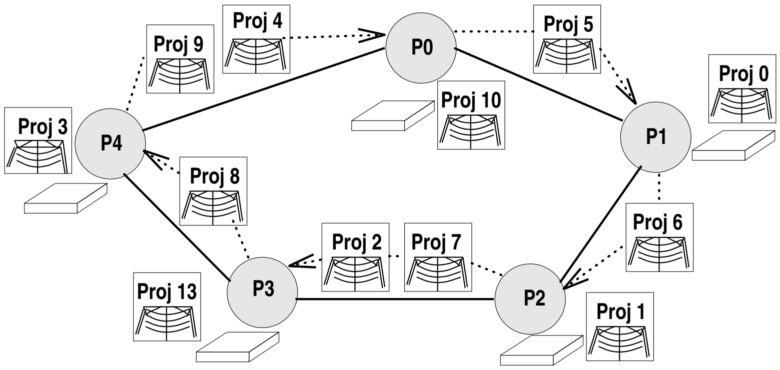
Asynchronous Implementation



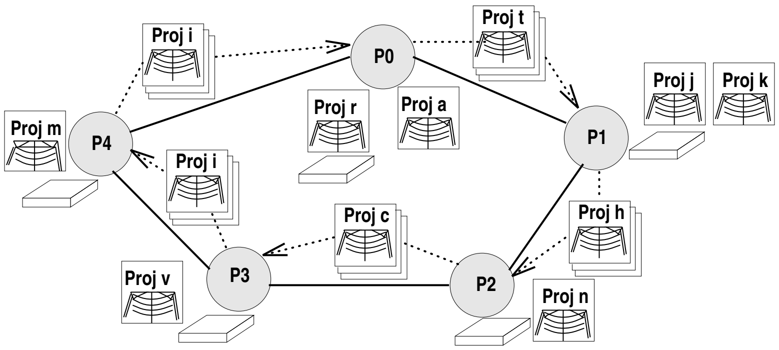
(g) Creation and local calculation step



(h) Communication, creation and local caculation step



(i) step +1



(j) step +n

Computations Times

Algorithms used are:

- Projection operator
- Backprojection operator
- Feldkamp Algorithm: Backprojection of the Filtered 2D Acquisitions

Implementations used are:

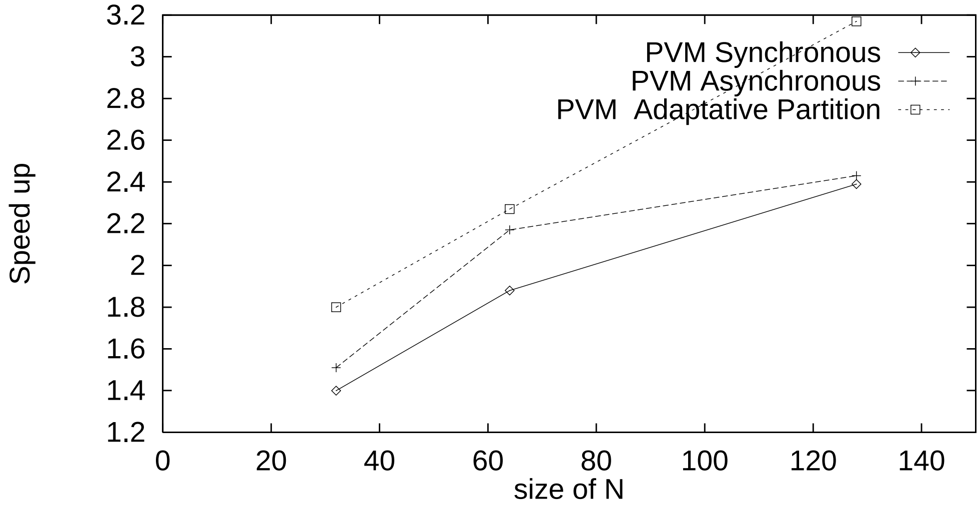
- Sequential mode for comparison
- Synchronous mode
- Asynchronous mode
- Asynchronous mode with Adaptative Data Partition

We tested for a N^3 Image ($N = 32, 64, 128$) and 100.128^2 Projections

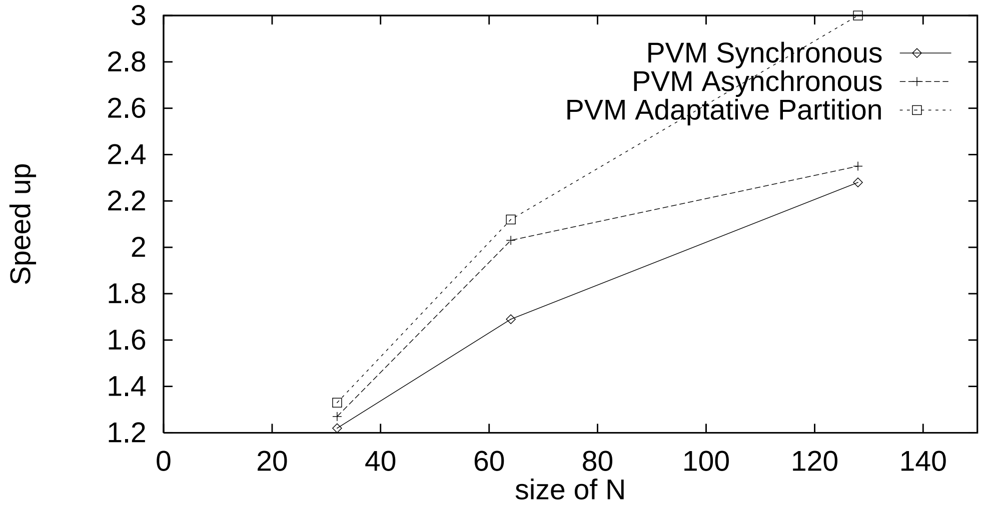
Some computation times (seconds) for $N = 128$...

Algorithm	PE	Projection	Backprojection	Feldkamp
Sequential	1	7639	7334	10606
Synchronous mode	5	3202	3211	3395
Asynchronous mode	5	3221	3088	3000
Adaptative data partition	5	2412	2449	2593
Maspar	1024	2486	4153	4276
Paragon	24	311	322	313

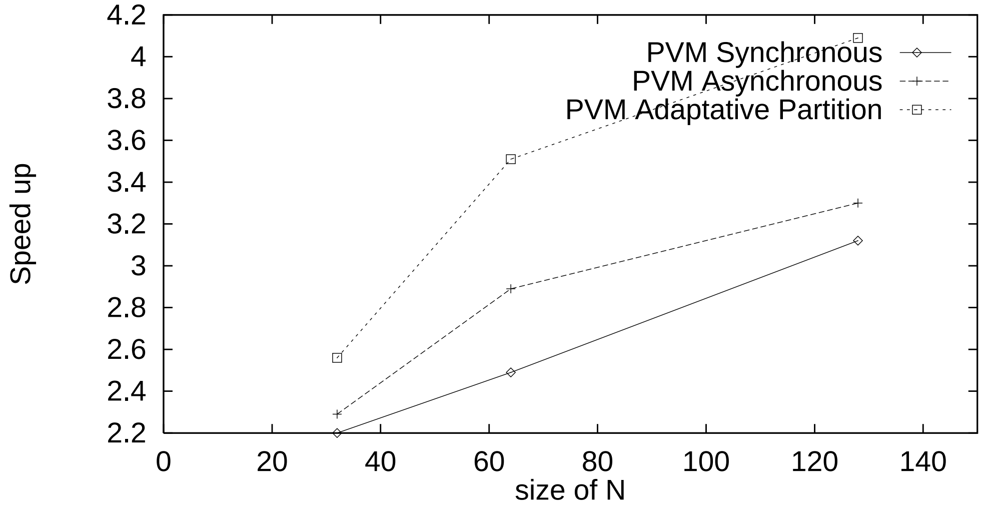
Speed up of different Implementations



(k) Projection Speed up

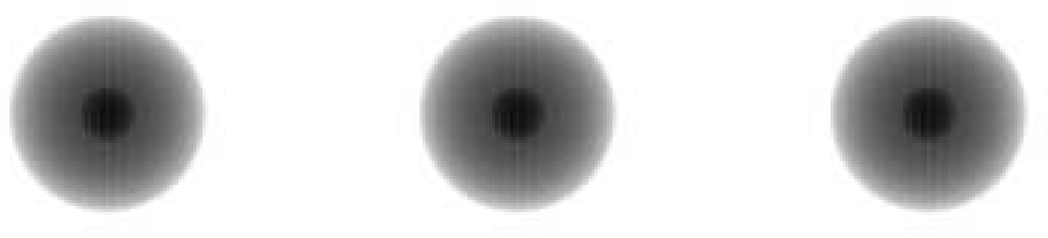


(l) Backprojection Speed up



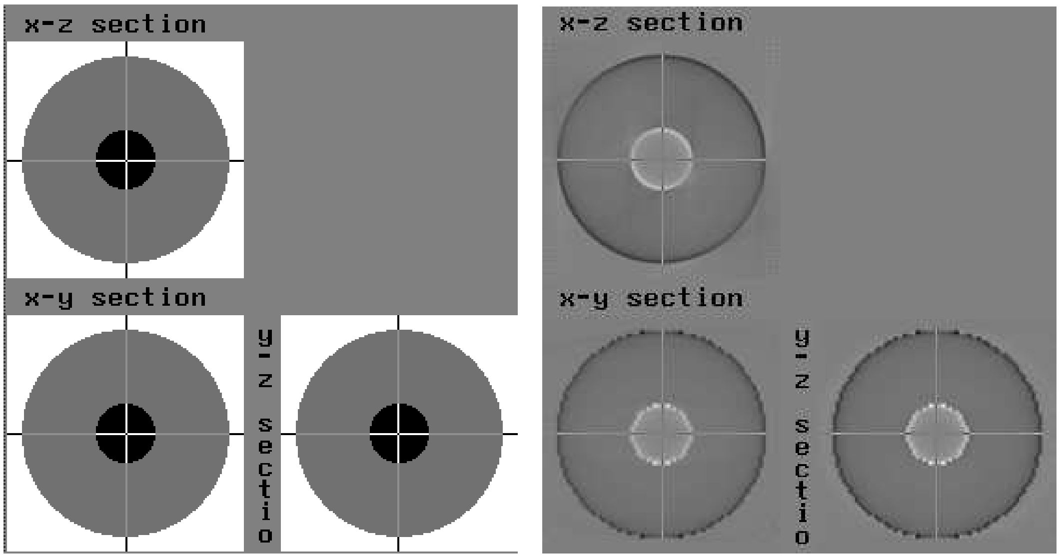
(m) Feldkamp Speed up

2D Projections and 3D Image



(n) $\Theta = 0$ (o) $\Theta = 60$ (p) $\Theta = 120$

Figure 1: PROJECTION OF 3D IMAGE AT DIFFERENT ANGLES OF VIEW (Θ)



(a) Initial 3D image (b) Reconstructed 3D image

Figure 2: VISUALISATION OF 3D IMAGE WITH THREE ORTHOGONAL SLICES

Future works

Large 3D Images $(256^3, 512^3)$

Experimental Data from the Morphometre

Implementation of Algebraic Methods

New Parallel Computer

- Cray T3D
- SP1 of IBM