ONE NEW APRIORI ESTIMATE IN THE THEORY OF VARIATIONAL INEQUALITIES VIA THE TECHNIQUES OF STOCHASTIC ANALYSIS

B. Dochviri, M. Shahiashvili

I. Vekua Institute of Applied Mathematics of Tbilisi State University

We consider the variational inequalities (in particular, obstacle problems) in a n-dimensional bounded smooth D for the elliptic operator

$$Au = \sum_{i,j=1}^{n} a_{ij}(x)\frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_{i=1}^{n} b_i(x)\frac{\partial u}{\partial x_i},$$

where $(a_{ij}(x))_{i,j=1,\dots,n}$ is a symmetric and uniformly positive definite matrix and $a_{ij}(x) \in C^1(\bar{D})$, $b_i(x) \in L^\infty(D)$.

For the initial data $g(x) \in H^{2,p}(D)$, $p > n$ and $c(x) \in L^\infty(D)$ find $u(x) \in H^{2,p}(D)$, $p > n$, such that

$$\begin{cases} Au + c \leq 0, \quad u - g \geq 0, \\ (Au + c)(u - g) = 0, \end{cases} \quad \textit{a.e. in } D, \ \textit{where } u - g \in H_0^1(D).$$

Let $u_i(x)$, $i = 1, 2$ be the corresponding solution of this variational inequality for the initial data $g_i(x)$, $c_i(x)$, $i = 1, 2$.

We prove the following inequality based on the techniques of stochastic analysis

$$\int_D |A(u_1(y) - u_2(y)) + (c_1(y) - c_2(y))| G_D(x,y)dy \leq$$
$$\leq \int_D |A(g_1(y) - g_2(y)) + (c_1(y) - c_2(y))| G_D(x,y)dy \quad \forall x \in D,$$

where $G_D(x,y)$ is a Green function for the operator A in the domain D.

This inequality leads to the following stability results: the value $\|u_1 - u_2\|_{H^2,P'(K)}$ can be estimated by corresponding values $\|g_1 - g_2\|_{H^2,P(D)}$ and $\|c_1 - c_2\|_{LP(D)}$, where $P' \in (1, P)$ is an arbitrary number from this interval and K is any fixed compact subset of D