STATIONARY HEAT DISTRIBUTION IN THE NONHOMOGENEOUS TRANSTROPIC PARABOLICAL CYLINDER

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In the cylinder-parabolical coordinate system ϱ, α, z, where $H_\varrho = H_\alpha = \sqrt{\varrho^2+\alpha^2}$, $H_z = 1$ are Lamé coeficients, the following differential equation

$$\lambda_1 \frac{\partial^2 T}{\partial \varrho^2} + \frac{\partial}{\partial \alpha}\left(\lambda_2 \frac{\partial T}{\partial \alpha}\right) + \left(\varrho^2+\alpha^2\right)\lambda_1 \frac{\partial^2 T}{\partial z^2} = 0 \tag{1}$$

is considered in the domain $\Omega = \{ -\infty < \varrho < \infty; 0 < \alpha_0 < \alpha < \alpha_1; z_0 < z < z_1 \}$.

Equation (1) discribes the stationary heat distribution in transtropic body, nonhomogeneous along α. In (1), $\lambda_2 = \lambda_2(\alpha)$ is heat conductivity along tangential to the coordinate line α, while $\lambda_1 = \lambda_1(\alpha)$ is heat conductivity along tangential to the coordinate line ϱ and coordinate line z.

Boundary conditions of the problem have the form

$$\begin{array}{ll} \text{For} \quad z = z_0 & T = 0, \\ \phantom{\text{For} \quad} z = z_1 & T = 0. \end{array} \tag{2}$$

$$\begin{array}{ll} \text{For} \quad \alpha = \alpha_0 & T = F_0(\varrho, z), \\ \phantom{\text{For} \quad} \alpha = \alpha_1 & T = F_1(\varrho, z). \end{array} \tag{3}$$

$$\begin{array}{lll} \text{For} \quad \varrho \to \infty & T \to 0 & \text{exponential convergence}, \\ \phantom{\text{For} \quad} \varrho \to -\infty & T \to 0 & \text{exponential convergence}. \end{array} \tag{4}$$

We assume that agreement conditions on the edges of the curvilinear coordinate parallelepiped hold.

Using the method of separation of variables we can write the solution of problem (1), (2), (3), (4) in the form:

$$T = \sum_{n=0}^{\infty}\sum_{k=1}^{\infty}\left(e^{-\frac{\pi k \varrho^2}{2 z_1}} H_n\left(\sqrt{\frac{\pi k}{z_1}}\varrho\right)\right) sin\left(\frac{\pi k}{z_1} z\right) A \tag{5}$$

H_n Heremit's number n polynomial, and A is the general solution of the following differential equation:

$$A'' + \frac{\lambda_2'}{\lambda_2} A' - \left(\mu + \theta^2\alpha^2\right)\frac{\lambda_1}{\lambda_2} A = 0 \tag{6}$$

where $\theta = \frac{\pi k}{z_1}$, $\mu = \theta(2n+1) = \frac{\pi k}{z_1}(2n+1)$.

General solution of (6) in the case under consideration ($\lambda_2 = e^{a\alpha}$, $\lambda_1 = g^2 e^{a\alpha}$; a and g are constants) can be presented by means of elementary functions. Thus, the general solution of (5) is presented as a product of trigonometric functions, Hermit polinomials and general solution of (6).

The investigated problem presents a mathematical model of a real object.

Convergence of the series and uniqueness of the solution are proved.