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A Note On Perfect Totient Numbers

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Abstract

In this note we prove that there are no perfect totient numbers of the form $3^k p$, $k \geq 4$, where $s = 2^a 3^b + 1$, $r = 2^c 3^d s + 1$, $q = 2^e 3^f r + 1$, and $p = 2^g 3^h q + 1$ are primes with $a, c, e, g \geq 1$, and $b, d, f, h \geq 0$.

1 Introduction

Let ϕ denote Euler's totient function. Define $\phi^1(n) = \phi(n)$ and $\phi^k(n) = \phi(\phi^{k-1}(n))$ for all integers $n > 2$, $k \geq 2$. Let c be the smallest positive integer such that $\phi^c(n) = 1$. Define the arithmetic function S by

$$S(n) = \sum_{k=1}^{c} \phi^k(n).$$

We say that n is a *perfect totient number* (or PTN for short) if $S(n) = n$.

There are infinitely many PTNs, since it is easy to show that 3^k is a PTN for all positive integers k. Perez Cacho [6] proved that $3p$, for an odd prime p, is a PTN if and only if $p = 4n + 1$, where n is a PTN. Mohan and Suryanarayana [5] proved that $3p$, for an odd prime p, is not a PTN if $p \equiv 3 \pmod 4$. Thus PTNs of the form $3p$ have been completely characterized. D. E. Iannucci, the author and G. L. Cohen [3] investigated PTNs of the form $3^k p$ in the following cases:

1. $k \geq 2$, $p = 2^c 3^d q + 1$ and $q = 2^a 3^b + 1$ are primes with $a, c \geq 1$ and b, $d \geq 0$;

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2. $k \geq 2$, $p = 2^e 3^f q + 1$, $q = 2^c 3^d r + 1$ and $r = 2^a 3^b + 1$ are all primes with a, c, $e \geq 1$ and b, d, $f \geq 0$;

3. $k \geq 3$, $p = 2^g 3^h q + 1$, $q = 2^e 3^f r + 1$, $r = 2^c 3^d s + 1$ and $s = 2^a 3^b + 1$, are all primes with $a, c, e, g \geq 1$, $b, d, f, h \geq 0$.

In the first case, they determined all PTNs for $k = 2, 3$ and proved that there are no PTNs of the form $3^k p$ for $k \geq 4$ by solving the related Diophantine equations. In the remaining cases, they only found several PTNs by computer searches. The author ([1, 2]) gave all solutions to the Diophantine equations $2^x - 2^y 3^z - 2 \cdot 3^u = 9^k + 1$, and $2^x - 2^y 3^z - 4 \cdot 3^w = 3 \cdot 9^k + 1$, which shows that there are no PTNs of the form $3^k p$ for $k \geq 4$ in the second case mentioned above.

In general, let$\mathcal{M}$ be the set of all perfect totients, I. E. Shparlinski [7] has shown that $\mathcal{M}$ is of asymptotic density zero, and F. Luca [4] showed that $\sum_{m \in \mathcal{M}} \frac{1}{m}$ converges.

The purpose of this note is to prove that, in the third case mentioned above, there are no PTNs of the form $3^k p$ for $k \geq 4$.

2 Lemmas

We first deduce related Diophantine equations. Let $k \geq 3$, $n = 3^k p$. Suppose all of $s = 2^a 3^b + 1$, $r = 2^c 3^d s + 1$, $q = 2^e 3^f r + 1$, and $p = 2^g 3^h q + 1$ are prime with $a, c, e, g \geq 1$, $b, d, f, h \geq 0$. If n is a PTN , then $S(n) = n$ by definition, which implies the diophantine equation

$$2^g(2^e(2^c(2^a - 3^{d+f+h+k-3}) - 3^{f+h+k-2}) - 3^{h+k-1}) = 3^k + 1. \tag{1}$$

Apparently, $g = 1$ or 2 for k even or odd, respectively. Next, according to $k = 2k_1$ or $k = 2k_1 + 1$, we consider more general Diophantine equations

$$2^x - 2^y 3^z - 2^u 3^v - 2 \cdot 3^w = 9^{k_1} + 1, \tag{2}$$

with $x \geq 4$, $y, u, w > 0$, $z, v \geq 0$, $k_1 \geq 2$, and

$$2^x - 2^y 3^z - 2^u 3^v - 4 \cdot 3^w = 3 \cdot 9^{k_1} + 1, \tag{3}$$

with $x \geq 4$, $y, u, w > 0$, $z, v \geq 0$, $k_1 \geq 1$, respectively. Since the terms $2^y 3^z$ and $2^u 3^v$ have symmetry in (2) and (3), we need only determine the solutions (x, y, z, u, v, w, k) to (2) and (3) such that $y > u$ or $y = u, z \geq v$.

Let (x, y, z, u, v, w, k_1) be any solution to the equation (2) (or (3)), and let

$$(x, y, z, u, v, w, k_1) \equiv (\alpha, \beta, \gamma, \delta, \lambda, \mu, \nu,) \pmod{36, 36, 36, 36, 36, 36, 18}$$

denote $x \equiv \alpha \pmod{36}, y \equiv \beta \pmod{36}, z \equiv \gamma \pmod{36}, u \equiv \delta \pmod{36}, v \equiv \lambda \pmod{36}$, $w \equiv \mu \pmod{36}$, and $k_1 \equiv \nu \pmod{18}$. In solving equation (2) and equation (3), we first determine all the $\alpha, \beta, \gamma, \delta, \lambda, \mu, \nu$.

Lemma 1. *Let (x, y, z, u, v, w, k_1) be any solution to the equation (2), and let*

$$(x, y, z, u, v, w, k_1) \equiv (\alpha, \beta, \gamma, \delta, \lambda, \mu, \nu) \pmod{36, 36, 36, 36, 36, 36, 18}.$$

Then all the possible $\alpha,\beta,\gamma,\delta,\lambda,\mu,\nu$ with $36\geq\alpha,\beta,\delta,\lambda\geq 1, 35\geq\gamma,\lambda\geq 0,\ 19\geq\nu\geq 2$, $\beta>\delta$ or $\beta=\delta$ and $\gamma\geq\lambda$ are listed in Table 1 and Table 1'.

PROOF: Since

$$2^{36}\equiv 1 \pmod{5\cdot 7\cdot 13\cdot 19\cdot 37\cdot 73}, 3^{36}\equiv 1 \pmod{5\cdot 7\cdot 13\cdot 19\cdot 37\cdot 73},$$

$\alpha,\beta,\gamma,\delta,\lambda,\mu,\nu$ must satisfy

$$2^\alpha-2^\beta 3^\gamma-2^\delta 3^\lambda-2\cdot 3^\mu\equiv 9^\nu+1 \pmod{5\cdot 7\cdot 13\cdot 19\cdot 37\cdot 73}. \tag{4}$$

But note that $2^x\equiv 0 \pmod{2^4}, 9^{k_1}\equiv 0 \pmod{3^3}, 2^{36}\equiv 1 \pmod{3^3}, 3^{36}\equiv 1 \pmod{2^4}$; $M=36l+m$ implies $2^M\equiv 0$ or $2^m \pmod{2^4}$ and $3^M\equiv 0$ or $3^m \pmod{3^3}$. Hence $\alpha,\beta,\gamma,\delta,\lambda,\mu,\nu$ must satisfy one of the 4 congruences

$$-2^\beta\cdot B\cdot 3^\gamma-2^\delta\cdot D\cdot 3^\lambda-2\cdot 3^\mu\equiv 9^\nu+1 \pmod{2^4} \tag{5}$$

and one of the 8 congruences

$$2^\alpha-2^\beta 3^\gamma\cdot C-2^\delta 3^\lambda\cdot E-2\cdot 3^\mu\cdot F\equiv 1 \pmod{3^3}, \tag{6}$$

where B,C,D,E,F take value $0,1$ independently. The congruences (4), (5) and (6) were tested on a computer with a program written in UBASIC. All the $(\alpha,\beta,\gamma,\delta,\lambda,\mu,\nu)$ that satisfy (4), (5) and (6) are divided into two parts: those listed in Table 1 are in fact solutions to equation (2), and the remainder, listed in Table 1', are not. □

Similarly, we have

LEMMA 2. *Let (x,y,z,u,v,w,k_1) be any solution to the equation (3), and let*

$$(x,y,z,u,v,w,k)\equiv(\alpha,\beta,\gamma,\delta,\lambda,\mu,\nu) \pmod{36,36,36,36,36,36,18}.$$

Then all the possible $\alpha,\beta,\gamma,\delta,\lambda,\mu,\nu$ with $36\geq\alpha,\beta,\delta,\mu\geq 1, 35\geq\gamma,\lambda\geq 0,\ 18\geq\nu\geq 1$, $\beta>\delta$ or $\beta=\delta$ and $\gamma\geq\lambda$ are listed in Table 2 and Table 2'.

LEMMA 3. *Let $(\alpha,\beta,\gamma,\delta,\lambda,\mu,\nu)$ be any solution to equation (2) or (3) that is listed in Table 1 or Table 2, and suppose*

1. *$\alpha>\beta>\delta$; or*
2. *$\alpha>\beta+2,\ \beta=\delta$;*

holds. Then there is no other solution $(x,y,\gamma,u,\lambda,\mu,\nu)$ to equation (2) or (3) that satisfies $(x,y,u)\equiv(\alpha,\beta,\delta) \pmod{36,36,36}$

PROOF: Let $x=\alpha+36i, y=\beta+36j, u=\delta+36l$. We have

$$2^\alpha(2^{36i}-1)=2^\beta 3^\gamma(2^{36j}-1)+2^\delta 3^\lambda(2^{36l}-1). \tag{7}$$

In case 1, consideration of (7), modulo 2^β and 2^α in turn gives $l=0$ and $j=0$. Hence we have $i=0$. In case 2, since $3^\gamma+3^\lambda\equiv 2,4 \pmod 8$, consideration of (7), modulo 2^α, gives $j=l=0$, and therefore $i=0$.

3 Main Results

THEOREM 1. *All the solutions to equation* (2) *are given by* $(x, y, z, u, v, w, k_1) = (\alpha, \beta, \gamma, \delta, \lambda, \mu, \nu)$ *with* $\alpha, \beta, \gamma, \delta, \lambda, \mu, \nu$ *listed in Table* 1.

PROOF: Let (x, y, z, u, v, w, k_1) be any solution to equation (2), and let

$$(x, y, z, u, v, w, k_1) \equiv (\alpha, \beta, \gamma, \delta, \lambda, \mu, \nu) \pmod{36, 36, 36, 36, 36, 36, 18}.$$

By Lemma 1, all of $\alpha, \beta, \gamma, \delta, \lambda, \mu, \nu$ are listed in Table 1 or Table 1′. Put $x = \alpha + 36i, y = \beta + 36j, z = \gamma + 36l, u = \delta + 36m, v = \lambda + 36n, w = \mu + 36t, k_1 = \nu + 18t_1$. Then we must have

$$2^{\alpha+36i} - 2^{\beta+36j} \cdot 3^{\gamma+36l} - 2^{\delta+36m} \cdot 3^{\lambda+36n} - 2 \cdot 3^{\mu+36t} \equiv 9^{\nu+36t_1} + 1 \pmod{11 \cdot 31 \cdot 181 \cdot 331 \cdot 631}. \quad (8)$$

For $\alpha, \beta, \gamma, \delta, \lambda, \mu, \nu$ appearing in Table 1, since $2^{180} \equiv 3^{360} \equiv 1 \pmod{11 \cdot 31 \cdot 181 \cdot 331 \cdot 631}$, we first test (8) within

$$4 \geq i \geq 0, 4 \geq j \geq 0, 9 \geq l \geq 0, 4 \geq m \geq 0, 9 \geq n \geq 0, 9 \geq t \geq 0, 9 \geq t_1 \geq 0.$$

With computer assistance, it follows that $l = n = t = t_1 = 0$ in this case. Since any $\alpha, \beta, \gamma, \delta, \lambda, \mu, \nu$ that listed in Table 1 satisfy the conditions of Lemma 3, we must have $i = j = m = 0$ by Lemma 3.

For $\alpha, \beta, \gamma, \delta, \lambda, \mu, \nu$ appearing in Table 1′, since $(\alpha, \beta, \gamma, \delta, \lambda, \mu, \nu)$ is not a solution to equation(2), we have $i \geq 1$. The congruence (8) was then tested on a computer within the ranges

$$5 \geq i \geq 1, 4 \geq j \geq 0, 9 \geq l \geq 0, 4 \geq m \geq 0, 9 \geq n \geq 0, 9 \geq t \geq 0, 9 \geq t_1 \geq 0$$

with no (i, j, l, m, n, t, t_1) being found, which shows that (x, y, z, u, v, w, k_1) cannot be a solution to equation (2). $\square$

THEOREM 2. *All the solutions to equation* (3) *are given by* $(x, y, z, u, v, w, k_1) = (\alpha, \beta, \gamma, \delta, \lambda, \mu, \nu)$ *with* $\alpha, \beta, \gamma, \delta, \lambda, \mu, \nu$ *listed in Table* 2 .

PROOF: The proof is basically the same as that for theorem 1, with the only difference being that $(\alpha, \beta, \gamma, \delta, \lambda, \mu, \nu) = (9, 7, 0, 7, 0, 1, 2)$, listed in Table 2, does not satisfy the conditions of Lemma 3. Suppose that $x = 9 + 36i, y = 7 + 36j, z = 36l, u = 7 + 36m, v = 36n, w = 1 + 36t, k_1 = 2 + 36t_1$ is a solution to equation (3). Then a computer test of the related congruence within $4 \geq i \geq 0, 4 \geq j \geq 0, 9 \geq l \geq 0, 4 \geq m \geq 0, 9 \geq n \geq 0, 9 \geq t \geq 0, 9 \geq t_1 \geq 0$ gives $l = n = t = t_1 = 0$. Consideration of (7) with $\alpha, \beta, \gamma, \delta, \lambda$ replaced by $9, 7, 0, 7, 0$, modulo 2^7, gives $j = l = 0$. Therefor $i = 0$.

THEOREM 3. *There are no PTNs of the form* $3^k p$, $k \geq 4$, *where all of* $s = 2^a 3^b + 1$, $r = 2^c 3^d s + 1$, $q = 2^e 3^f r + 1$, *and* $p = 2^g 3^h q + 1$ *are prime with* $a, c, e, g \geq 1$, $b, d, f, h \geq 0$.

PROOF: Suppose (a, c, d, e, f, g, h, k) is a solution to equation (1). Let $x = a + c + e + g$, $y = c + e + g, z = d + f + k - 3, u = e + g, v = f + h + k - 2, w = h + k - 1$, and $k_1 = \frac{k}{2}$ or $k_1 = \frac{k-1}{2}$ for k even or odd, respectively. Then (x, y, z, u, v, w, k_1) must be a solution to equation (2) or equation (3). From the first two theorems it follows that the only solutions to equation(1) are $(a, c, d, e, f, h, k) = (4, 1, 0, 1, 2, 1, 3)$,$(1, 2, 0, 4, 0, 0, 3)$,$(3, 1, 0, 4, 1, 0, 3)$,$(2, 2, 1, 4, 0, 0, 3)$, $(8, 1, 4, 1, 0, 1, 3)$,$(5, 1, 2, 4, 1, 0, 3)$,$(4, 2, 0, 4, 2, 0, 3)$. $\square$

α	β	γ	δ	λ	μ	ν	α	β	γ	δ	λ	μ	ν	α	β	γ	δ	λ	μ	ν
7	2	2	2	0	1	2	10	5	3	3	1	3	2	12	8	2	6	2	5	3
7	3	1	2	0	2	2	10	5	3	3	2	1	2	12	10	1	5	2	1	3
7	4	0	2	1	2	2	10	6	1	4	1	3	3	13	6	0	2	3	6	4
7	4	0	3	1	1	2	10	6	1	5	1	1	3	13	8	3	6	0	5	3
7	5	0	3	0	1	2	10	7	0	2	0	4	3	13	10	1	4	0	7	3
8	1	1	1	1	4	2	10	7	1	3	2	5	2	14	6	5	4	1	3	3
8	1	4	1	1	1	2	10	8	0	5	0	1	3	14	6	5	5	1	1	3
8	2	3	2	1	3	2	10	8	1	2	1	4	2	14	10	1	2	3	8	2
8	3	0	2	0	4	2	11	3	5	2	0	2	2	15	10	3	4	0	7	3
8	4	1	2	3	2	2	11	4	0	3	5	1	2	16	3	6	3	4	1	5
8	4	1	3	2	3	2	11	4	4	2	0	2	3	16	5	4	4	5	1	5
8	4	2	2	1	2	2	11	4	4	4	0	1	3	16	6	4	4	4	1	5
8	4	2	3	1	1	2	11	7	2	2	0	4	3	16	7	1	6	3	7	5
8	5	1	3	1	3	2	11	8	0	6	2	5	3	16	8	5	5	4	1	3
8	5	1	3	2	1	2	11	8	1	6	0	5	3	16	9	1	6	2	7	5
9	8	0	2	1	4	2	11	10	0	5	2	1	3	16	9	4	3	7	1	4
10	3	1	2	3	4	3	12	4	3	2	6	2	3	16	9	4	5	6	1	3
10	3	3	3	1	3	3	12	4	5	2	3	2	2	16	11	0	6	0	7	5
10	3	3	3	2	1	3	12	4	5	3	2	3	2	16	11	1	2	4	2	5
10	4	2	4	2	1	3	12	5	2	2	6	4	3	16	11	1	5	2	3	5
10	4	3	3	1	5	2	12	5	4	5	2	5	3	16	13	1	5	3	9	3
10	5	1	2	2	4	3	12	7	2	6	3	5	3	18	10	5	2	3	8	2
10	5	1	4	2	3	3	12	7	3	3	2	5	2	18	13	3	5	3	9	3
10	5	2	3	4	1	2	12	8	1	5	4	1	3							

Table 1

α	β	γ	δ	λ	μ	ν	α	β	γ	δ	λ	μ	ν	α	β	γ	δ	λ	μ	ν
5	1	0	1	0	2	19	6	3	1	2	1	2	19	8	3	14	2	1	16	13
5	2	1	2	0	1	19	6	3	1	2	2	36	18	8	3	15	2	1	14	13
5	3	0	2	0	2	18	6	3	12	2	2	14	12	8	6	1	2	2	2	19
5	3	1	2	0	36	18	6	3	13	2	2	12	12	10	3	1	2	5	2	19
5	3	12	2	0	14	12	6	4	0	2	2	36	19	10	9	0	3	1	5	18
5	4	0	20	0	36	19	6	5	0	2	0	2	19	12	7	2	2	6	2	19
5	3	13	2	0	12	12	7	5	1	2	0	2	19	18	34	22	21	16	33	11
6	2	2	2	1	1	19	8	3	2	2	1	4	19	30	23	2	1	13	1	15
6	3	0	2	2	2	18	8	3	3	2	1	2	19							

Table 1´

α	β	γ	δ	λ	μ	ν	α	β	γ	δ	λ	μ	ν	α	β	γ	δ	λ	μ	ν
6	1	2	1	1	1	1	10	6	2	5	1	4	1	12	7	3	7	1	1	2
6	2	1	2	1	1	1	10	6	2	6	1	1	2	12	8	1	7	1	6	1
6	4	0	3	0	1	1	10	7	1	3	2	4	2	12	8	2	6	2	5	2
7	4	0	3	2	1	1	10	7	1	5	2	3	2	12	8	2	6	3	2	1
7	4	1	4	0	2	1	10	7	1	5	2	4	1	12	9	1	4	1	4	3
7	5	0	5	0	2	1	10	7	1	6	2	2	1	12	9	1	8	2	1	2
7	6	0	3	1	1	1	10	7	1	7	1	1	2	12	10	0	7	0	6	1
8	1	4	1	3	1	1	10	8	1	3	3	1	1	12	10	1	3	1	5	1
8	2	3	2	1	3	1	10	8	1	6	1	2	1	12	10	1	8	1	1	2
8	2	3	2	3	1	1	10	9	0	8	0	1	2	13	6	0	3	6	3	3
8	4	1	3	2	3	1	11	6	0	3	5	1	1	13	6	0	5	5	3	2
8	4	2	3	2	1	1	11	6	3	6	0	1	2	13	6	0	5	5	4	1
8	4	2	4	1	2	1	11	8	0	6	2	5	2	13	6	4	6	0	6	1
8	5	1	3	1	3	1	11	8	0	6	3	2	1	13	8	3	6	0	5	2
8	5	1	5	1	2	1	11	8	1	6	0	5	2	13	10	0	8	3	1	2
8	6	1	3	1	1	1	11	9	1	8	0	1	2	13	10	1	4	0	6	3
8	7	0	6	0	2	1	11	10	0	3	1	5	1	13	12	0	7	2	6	1
9	4	0	3	3	2	2	11	10	0	8	1	1	2	14	6	5	6	2	1	2
9	4	2	4	0	3	2	12	4	5	3	2	3	1	14	8	1	6	5	2	1
9	4	2	4	0	4	1	12	4	5	4	2	2	1	14	9	3	4	1	4	3
9	4	3	4	0	2	1	12	5	2	3	4	5	3	14	9	3	8	2	1	2
9	6	0	5	1	3	2	12	5	2	3	4	6	2	14	10	1	7	4	6	1
9	6	0	5	1	4	1	12	5	2	4	4	4	3	14	10	2	8	3	1	2
9	6	1	6	0	1	2	12	5	3	3	2	5	3	14	12	1	7	2	6	1
9	7	0	5	0	3	2	12	5	3	3	2	6	2	15	10	3	4	0	6	3
9	7	0	5	0	4	1	12	5	3	5	2	6	1	16	8	5	7	1	6	1
9	7	0	7	0	1	2	12	5	4	5	2	5	2	16	10	1	8	5	1	2
9	7	1	6	0	2	1	12	6	2	4	4	2	3	18	4	3	3	6	9	5
9	8	0	3	3	1	1	12	6	2	6	2	6	1	18	4	3	3	6	10	4
9	8	0	6	1	2	1	12	6	3	3	2	3	3	18	5	6	4	3	10	3
10	1	2	1	1	5	1	12	6	3	4	2	2	3	18	5	6	5	4	10	1
10	2	1	2	1	5	1	12	7	2	3	4	3	3	18	7	4	6	5	10	1
10	2	5	2	1	1	1	12	7	2	3	5	5	1	18	8	4	6	4	10	1
10	3	4	3	1	3	2	12	7	2	4	3	4	3	18	10	3	7	5	8	5
10	3	4	3	1	4	1	12	7	2	5	4	3	2	18	10	4	3	5	3	5
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10	5	3	3	1	3	1	12	7	3	5	2	3	2	18	11	4	3	7	9	1
10	5	3	5	1	2	1	12	7	3	5	2	4	1	18	11	4	5	7	8	1
10	6	2	5	1	3	2	12	7	3	6	2	2	1	18	11	4	7	6	6	1

Table 2

α	β	γ	δ	λ	μ	ν	α	β	γ	δ	λ	μ	ν	α	β	γ	δ	λ	μ	ν
6	1	2	1	1	2	18	7	6	0	3	1	2	18	18	18	18	2	9	1	14
6	2	1	2	1	2	18	8	3	3	3	1	1	18	18	18	20	1	14	31	18
6	2	2	2	1	1	18	8	5	1	4	2	1	18	18	20	33	14	20	36	1
6	3	1	3	1	1	18	8	6	1	3	1	2	18	18	23	12	17	3	32	1
6	3	35	3	1	2	17	8	6	1	4	1	1	18	18	27	30	3	34	9	7
6	4	0	3	0	2	18	8	9	35	3	2	1	17	18	28	18	7	20	3	2
6	5	35	4	0	2	17	10	3	1	3	1	5	18	18	28	18	7	20	4	1
6	6	35	3	35	2	18	12	7	2	3	1	6	18	18	29	2	3	29	31	6
6	6	35	4	35	2	17	18	11	0	10	4	36	5	18	29	5	23	21	34	3
6	9	33	3	0	2	16	18	18	18	2	1	9	14	18	33	35	19	24	12	15

Table 2´

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References

[1] Moujie Deng, On the Diophantine equation $2^x - 2^y 3^z - 2 \cdot 3^u = 9^k + 1$, (in Chinese), *Journal of Natural Science of Heilongjiang University*, **23** (2006), 87–91.

[2] Moujie Deng, On the Diophantine equation $2^x - 2^y 3^z - 4 \cdot 3^w = 3 \cdot 9^k + 1$,(in Chinese), *Journal of Inner Mongolia Normal University*, **37** (2008), 45–49.

[3] D. E. Iannucci, D. Moujie and G. L. Cohen, On perfect totient numbers, *J. Integer Sequences* **6** (2003), Article 03.4.5.

[4] F. Luca, On the distribution of perfect totients, *J. Integer Sequences* **9** (2006), Article 06.4.4.

[5] A. L. Mohan and D. Suryanarayana, "Perfect totient numbers", in: *Number Theory (Proc. Third Matscience Conf., Mysore, 1981)* Lect. Notes in Math. **938**, Springer-Verlag, New York, 1982, 101–105.

[6] L. Perez Cacho, Sobre la suma de indicadores de ordenes sucesivos, *Revista Matematica Hispano-Americana* **5.3** (1939), 45–50.

[7] I. E. Shparlinski, On the sum of iterations of the Euler function, *J. Integer Sequences* **9** (2006), Article 06.1.6.

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