Electronic Transactions on Numerical Analysis.
Volume 37, pp. 189-201, 2010.

ISSN 1068-9613.

ETNA
Kent State University
http://etna.math.kent.edu

NEW QUADRILATERAL MIXED FINITE ELEMENTS*

YUNKYONG HYON † AND DO Y. KWAK‡

Abstract. In this paper, we introduce a new family of mixed finite element spaces of higher order ($k \geq 1$) on general quadrilateral grids. A typical element has two fewer degrees of freedom than the well-known Raviart-Thomas finite element $RT_{[k]}$, yet enjoys an optimal-order approximation for the velocity in L^2-norm. The order of approximation in the divergence norm is one less than the velocity, as is common to all other known elements, except for a recent element introduced by Arnold et al. [SIAM J. Numer. Anal., 42 (2005), pp. 2429–2451]. However, we introduce a local post-processing technique to obtain an optimal order in L^2-norm of divergence. This technique can be used to enhance the result of $RT_{[k]}$ element as well, and hence, can be easily incorporated into existing codes.

Our element has one lower order of approximation in pressure than the $RT_{[k]}$ element. However, the pressure also can be locally post-processed to produce an optimal-order approximation. The greatest advantage of our finite element lies in the fact that it has the fewest degrees of freedom among all the known quadrilateral mixed finite elements and thus, together with the post-processing techniques, provides a very efficient way of computing flow variables in mixed formulation. Numerical examples are in quite good agreement with the theory even for the case of almost degenerate quadrilateral grids.

Key words. Mixed finite element method, quadrilateral grid, optimal velocity, post-processing.

AMS subject classifications. 65N15, 65N30

* Received December 19, 2007. Accepted for publication December 22, 2009. Published online June 18, 2010. Recommended by S. Brenner.

†Institute for Mathematics and Its Applications, University of Minnesota, 114 Lind, 207 Church St. S.E., Minneapolis, MN 55455, USA, email: hyon@ima.umn.edu

‡Department of Mathematical Sciences, Korea Advanced Institute of Science and Technology, Daejeon, Korea 305-701. email:kdy@kaist.ac.kr