Electronic Transactions on Numerical Analysis
Volume 16, 2003

Contents

Abstract.

In this paper we are concerned with the spectral analysis of the sequence of preconditioned matrices $\{P_n^{-1}A_n(a, m_1, m_2, k)\}_n$, where $n = (n_1, n_2)$, $N(n) = n_1 n_2$ and where $A_n(a, m_1, m_2, k) \in \mathbb{R}^{N(n)\times N(n)}$ is the symmetric two-level matrix coming from a high–order Finite Difference (FD) discretization of the problem

$$\begin{cases} (-1)^k \left(\dfrac{\partial^k}{\partial x^k}\left(a(x,y)\dfrac{\partial^k}{\partial x^k}u(x,y)\right) + \dfrac{\partial^k}{\partial y^k}\left(a(x,y)\dfrac{\partial^k}{\partial y^k}u(x,y)\right)\right) = f(x,y), \\ \text{on } \Omega = (0,1)^2, \\ \left(\dfrac{\partial^s}{\partial \nu^s}u(x,y)\right)_{|\partial\Omega} = 0 \quad s = 0,\ldots,k-1 \quad \text{on } \partial\Omega, \end{cases}$$

with ν denoting the unit outward normal direction and where m_1 and m_2 are parameters identifying the precision order of the used FD schemes. We assume that the coefficient $a(x,y)$ is nonnegative and that the set of the possible zeros can be represented by a finite collection of curves. The proposed preconditioning matrix sequences correspond to two different choices: the Toeplitz sequence $\{A_n(1, m_1, m_2, k)\}_n$ and a Toeplitz based sequence that adds to the Toeplitz structure the informative content given by the suitable scaled diagonal part of $A_n(a, m_1, m_2, k)$. The former case gives rise to optimal preconditioning sequences under the assumption of positivity and boundedness of a. With respect to the latter, the main result is the proof of the asymptotic clustering at unity of the eigenvalues of the preconditioned matrices, where the "strength" of the cluster depends on the order k, on the regularity features of $a(x,y)$ and on the presence of zeros of $a(x,y)$.

Key Words.

finite differences, Toeplitz and Vandermonde matrices, clustering and preconditioning, spectral distribution.

AMS(MOS) Subject Classifications.

65F10, 65N22, 65F15.

Files.
vol.16.2003/pp1-29.dir/pp1-29.ps;
vol.16.2003/pp1-29.dir/pp1-29.pdf;

Forward References.

30 Vaidya's Preconditioners: implementation and experimental study. *Doron Chen and Sivan Toledo.*

Abstract.

We describe the implementation and performance of a novel class of preconditioners. These preconditioners were proposed and theoretically analyzed by Pravin Vaidya in 1991, but no report on their implementation or performance in practice has ever been published. We show experimentally that these preconditioners have some remarkable properties. We show that within the class of diagonally-dominant symmetric matrices, the cost and convergence of these preconditioners depends almost only on the nonzero structure of the matrix, but not on its numerical values. In particular, this property leads to robust convergence behavior on difficult 3-dimensional problems that cause stagnation in incomplete-Cholesky preconditioners (more specifically, in drop-tolerance incomplete Cholesky without diagonal modification, with diagonal modification, and with relaxed diagonal modification). On such problems, we have observed cases in which a Vaidya-preconditioned solver is more than times faster than an incomplete-Cholesky-preconditioned solver, when we allow similar amounts of fill in the factors of both preconditioners. We also show that Vaidya's preconditioners perform and scale similarly or better than drop-tolerance relaxed-modified incomplete Cholesky preconditioners on a wide range of 2-dimensional problems. In particular, on anisotropic 2D problems, Vaidya's preconditioners deliver robust convergence independently of the direction of anisotropy and the ordering of the unknowns. However, on many 3D problems in which incomplete-Cholesky-preconditioned solvers converge without stagnating, Vaidya-preconditioned solvers are much slower. We also show how the insights gained from this study can be used to design faster and more robust solvers for some difficult problems.

Key Words.

linear-equation solvers, iterative solvers, preconditioning, support preconditioning, support theory, maximum-spanning trees, experimental study.

AMS(MOS) Subject Classifications.

65-05, 65F10, 65F35, 65F50, 65N22, 05C05, 05C50, 05C85.

Files.
vol.16.2003/pp30-49.dir/pp30-49.ps;
vol.16.2003/pp30-49.dir/pp30-49.pdf;

Forward References.

50 General theorems for numerical approximation of stochastic processes on the Hilbert space. *Henri Schurz.*

Abstract.

General theorems for the numerical approximation on the separable Hilbert space of cadlag, -adapted stochastic processes with -integrable second moments is presented for nonrandom intervals and positive measure. The use of the theorems is illustrated by the special case of systems of ordinary stochastic differential equations (SDEs) and their numerical approximation given by the drift-implicit Euler method under one-sided Lipschitz-type conditions.

Key Words.

stochastic-numerical approximation, stochastic Lax-Theorem, ordinary stochastic differential equations, numerical methods, drift-implicit Euler methods, balanced implicit methods.

AMS(MOS) Subject Classifications.

65C20, 65C30, 65C50, 60H10, 37H10, 34F05.

Files.
vol.16.2003/pp50-69.dir/pp50-69.ps;
vol.16.2003/pp50-69.dir/pp50-69.pdf;

Forward References.

Abstract.

This paper introduces a new fast algorithm for uniform-resolution filtering of functions defined on the sphere. We use a fast summation algorithm based on Nonequispaced Fast Fourier Transforms, building on previous work that used Fast Multipole Methods. The resulting algorithm performs a triangular truncation of the spectral coefficients while avoiding the need for fast spherical Fourier transforms. The method requires operations for grid points. Furthermore, we apply these techniques to obtain a fast wavelet decomposition algorithm on the sphere. We present the results of numerical experiments to illustrate the performance of the algorithms.

Key Words.

spherical filter, spherical Fourier transform, spherical harmonics, associated Legendre functions, fast discrete transforms, fast Fourier transform at nonequispaced knots, wavelets, fast discrete summation.

AMS(MOS) Subject Classifications.

65Txx, 33C55, 42C10.

Files.
vol.16.2003/pp70-93.dir/pp70-93.ps;
vol.16.2003/pp70-93.dir/pp70-93.pdf;

Forward References.

Abstract.

In this paper we apply a minmax characterization for nonoverdamped nonlinear eigenvalue problems to a rational eigenproblem governing mechanical vibrations of a tube bundle immersed in an inviscid compressible fluid. This eigenproblem is nonstandard in two respects: it depends rationally on the eigenparameter, and it involves non-local boundary conditions. Comparison results are proved comparing the eigenvalues of the rational problem to those of certain linear problems suggesting a way how to construct ansatz vectors for an efficient projection method.

Key Words.

nonlinear eigenvalue problem, maxmin principle, fluid structure interaction.

AMS(MOS) Subject Classifications.
49G05.

Files.
vol.16.2003/pp94-106.dir/pp94-106.ps;
vol.16.2003/pp94-106.dir/pp94-106.pdf;

Forward References.

Abstract.

A new parameter choice method for Tikhonov regularization of discrete ill-posed problems is presented. Some of the regularized solutions of a discrete ill-posed problem are less sensitive than others to the perturbations in the right-hand side vector. This method chooses one of the insensitive regularized solutions using a certain criterion. Numerical experiments show that the new method is competitive with the popular L-curve method. An analysis of the new method is given for a model problem, which explains how this method works.

Key Words.

discrete ill-posed problems, discrete Picard condition, Tikhonov regularization.

AMS(MOS) Subject Classifications.

65F22.

Files.
vol.16.2003/pp107-128.dir/pp107-128.ps;
vol.16.2003/pp107-128.dir/pp107-128.pdf;

Forward References.

Abstract.

We present a new block method for solving large nonsymmetric linear systems of equations with multiple right-hand sides. We first give the matrix polynomial interpretation of the classical block biconjugate gradient (Bl-BCG) algorithm using formal matrix-valued orthogonal polynomials. This allows us to derive a block version of BiCGSTAB. Numerical examples and comparisons with other block methods are given to illustrate the effectiveness of the proposed method.

Key Words.

block Krylov subspace, block methods, Lanczos method, multiple right-hand sides, nonsymmetric linear systems.

AMS(MOS) Subject Classifications.

65F10.

Files.
vol.16.2003/pp129-142.dir/pp129-142.ps;
vol.16.2003/pp129-142.dir/pp129-142.pdf;

Forward References.

143 A quadrature formula of rational type for integrands with one endpoint singularity. *J. Illán.*

Abstract.

The paper deals with the construction of an efficient quadrature formula of rational type to evaluate the integral of functions which are analytic in the interval of integration, except at the endpoints. Basically our approach consists in introducing a change of variable u_q into the integral $I(f,h,r)$

$$I(f,h,r) = \int_{-(1-h)}^{(1-h)r} f(x)dx = \int_{\mu_q}^{\rho_q} F(u_q(x))u_q'(x)dx = I(f,q,h,r),$$

where $f \in H^p$ and $u_q(x) = w_a^q(x) = w_a(w_a^{q-1}(x))$, $w_a(z) = (z-a)/(1-az)$, $0 < a < 1$.
We evaluate the new form $I(f,q,h,r)$ by a quadrature approximant $Q_n(f) = Q(f,n,q,h,r,a)$ which is based on Hermite interpolation by means of rational functions. The nodes of $Q_n(f)$ are derived from a fundamental result proved by Ganelius [Anal. Math., 5 (1979), pp. 19-33] in connection with the problem of approximating the function $f_\alpha(x) = x^\alpha$, $0 \le x \le 1$, by means of rational functions.
We find (a_n) such that $Q_n(f) \to I(f,r) = I(f,0,r)$ as $h_n = \epsilon(1-a_n) \to 0$, for all $f \in H^p$. For functions in H^p, $1 < p < \infty$, which satisfy an integral Lipschitz condition of order β, the following estimate is deduced

$$E_n(f) = |I(f,r) - Q_n(f)| \le M\sqrt{n}\,\exp\left(-\pi\sqrt{n\beta(2q-1-1/p)}\right).$$

If $\beta = q = 1$ then the upper bound for $E_n(f)$ is that which is exact for the optimal quadrature error in H^p, $p > 1$.
We report some numerical examples to illustrate the behavior of the method for several values of the parameters.

Key Words.

interpolatory quadrature formulas, rational approximation, order of convergence, boundary singularities.

AMS(MOS) Subject Classifications.

41A25, 41A55, 65D30, 65D32.

Files.
vol.16.2003/pp143-164.dir/pp143-164.ps;
vol.16.2003/pp143-164.dir/pp143-164.pdf;

Forward References.

165 Analysis of two-dimensional FETI-DP preconditioners by the standard additive Schwarz framework. *Susanne C. Brenner.*

Abstract.
FETI-DP preconditioners for two-dimensional elliptic boundary value problems with heterogeneous coefficients are analyzed by the standard additive Schwarz framework. It is shown that the condition number of the preconditioned system for both second order and fourth order problems is bounded by $C\big(1+\ln(H/h)\big)^2$, where H is the maximum of the diameters of the subdomains, h is the mesh size of a quasiuniform triangulation, and the positive constant C is independent of h, H, the number of subdomains and the coefficients of the boundary value problems on the subdomains. The sharpness of the bound for second order problems is also established.

Key Words.
FETI-DP, additive Schwarz, domain decomposition, heterogeneous coefficients.

AMS(MOS) Subject Classifications.
65N55, 65N30.

Files.
vol.16.2003/pp165-185.dir/pp165-185.ps;
vol.16.2003/pp165-185.dir/pp165-185.pdf;

Forward References.

186 Gradient method with dynamical retards for large-scale optimization problems. *Francisco Luengo and Marcos Raydan.*

Abstract.

We consider a generalization of the gradient method with retards for the solution of large-scale unconstrained optimization problems. Recently, the gradient method with retards was introduced to find global minimizers of large-scale quadratic functions. The most interesting feature of this method is that it does not involve a decrease in the objective function, which allows fast local convergence. On the other hand, nonmonotone globalization strategies, that preserve local behavior for the non-quadratic case, have proved to be very effective when associated with low storage methods. In this work, the gradient method with retards is generalized and combined in a dynamical way with nonmonotone globalization strategies to obtain a new method for minimizing nonquadratic functions, that can deal efficiently with large problems. Encouraging numerical experiments on well-known test problems are presented.

Key Words.

spectral gradient method, nonmonotone line search, Barzilai-Borwein method, Polak-Ribière method, Rayleigh quotient.

AMS(MOS) Subject Classifications.

49M07, 49M10, 65K.

Files.
vol.16.2003/pp186-193.dir/pp186-193.ps;
vol.16.2003/pp186-193.dir/pp186-193.pdf;

Forward References.