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Contents

Abstract.
A singularly perturbed convection-diffusion problem is considered. The problem is discretized using an inverse-monotone finite volume method on Shishkin meshes. We establish first-order convergence in a global energy norm and a mesh-dependent discrete energy norm, no matter how small the perturbation parameter. Numerical experiments support the theoretical results.

Key Words.
convection-diffusion problems, finite volume methods, singular perturbation, Shishkin mesh.

AMS(MOS) Subject Classifications.
65N30.

Files.
vol.13.2002/pp1-11.dir/pp1-11.ps;
vol.13.2002/pp1-11.dir/pp1-11.pdf;

Forward References.

Abstract.
We study the asymptotic distribution of general interpolation arrays for a large class of even exponential weights on the line and $(-1, 1)$. Our proofs rely on deep properties of logarithmic potentials. We conclude with some open problems.

Key Words.
asymptotic distribution, Freud weight, Erds weight, exponential weight, interpolation, Lebesgue constant, logarithmic potential, Pollaczek weight, sup norm, weighted approximation.

AMS(MOS) Subject Classifications.
42C15, 42C05, 65D05.

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Forward References.

Abstract.
Some very effective but expensive image reconstruction algorithms cannot be applied to large images because of their cost. In this work, we first show how to apply such algorithms to subimages, giving improved reconstruction of regions of interest. Our second contribution is to construct confidence intervals for pixel values, by generalizing a theorem of O'Leary and Rust to allow both upper and lower bounds on variables. All current algorithms for image deblurring or deconvolution output an image. This provides an estimated value for each pixel in the image. What is lacking is an estimate of the statistical confidence that we can have in those pixel values or in the features they form in the image. There are two obstacles in determining confidence intervals for pixel values: first, the process is computationally quite intensive, and second, there has been no proposal for providing the results in a visually useful way. In this work we overcome the first of those limitations and develop an algorithm called Twinkle to overcome the second. We demonstrate the usefulness of these techniques on astronomical and motion-blurred images.

Key Words.
image restoration, regularization, confidence intervals, confidence images, motion blur, conjugate gradient method.

AMS(MOS) Subject Classifications.
65F10, 65F20, 65F30.

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vol.13.2002/pp22-37.dir/pp22-37.ps;
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Abstract.
This paper deals with parallel asynchronous linear iterations perturbed by errors in floating point arithmetic. An original result is presented which permits one to localize the limits of perturbed parallel asynchronous linear iterations. The result is established by using the approximate contraction concept. Simple examples are studied.

Key Words.
approximate contraction, parallel algorithms, asynchronous iterations.

AMS(MOS) Subject Classifications.
65F10, 65G05, 65Y05, 68Q22, 68Q10.

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Forward References.

56 On Error Estimation in the Conjugate Gradient Method And Why It Works In Finite Precision Computations. *Zdeněk Strakoš Pod Vodárenskou věží.*

Abstract.
In their paper published in 1952, Hestenes and Stiefel considered the conjugate gradient (CG) method an iterative method which terminates in at most n steps if no rounding errors are encountered [p. 410]. , They also proved identities for the A-norm and the Euclidean norm of the error which could justify the stopping criteria [Theorems 6:1 and 6:3, p. 416]. , The idea of estimating errors in iterative methods, and in the CG method in particular, was independently (of these results) promoted by Golub; the problem was linked to Gauss quadrature and to its modifications [7], [8]. A comprehensive summary of this approach was given in [15], [16]. During the last decade several papers developed error bounds algebraically without using Gauss quadrature. However, we have not found any reference to the corresponding results in [24]. All the existing bounds assume exact arithmetic. Still they seem to be in a striking agreement with finite precision numerical experiments, though in finite precision computations they estimate quantities which can be orders of magnitude different from their exact precision counterparts! For the lower bounds obtained from Gauss quadrature formulas this nontrivial phenomenon was explained, with some limitations, in [17].
In our paper we show that the lower bound for the A-norm of the error based on Gauss quadrature ([15], [17], [16]) is mathematically equivalent to the original formula of Hestenes and Stiefel [24]. We will compare existing bounds and we will demonstrate necessity of a proper rounding error analysis: we present an example of the well-known bound which can fail in finite precision arithmetic. We will analyse the simplest bound based on [24, Theorem 6:1], and prove that it is numerically stable. Though we concentrate mostly on the lower bound for the A-norm of the error, we describe also an estimate for the Euclidean norm of the error based on [24, Theorem 6:3]. Our results are illustrated by numerical experiments.

Key Words.
conjugate gradient method, Gauss quadrature, evaluation of convergence, error bounds, finite precision arithmetic, rounding errors, loss of orthogonality.

AMS(MOS) Subject Classifications.
15A06, 65F10, 65F25, 65G50.

Files.
vol.13.2002/pp56-80.dir/pp56-80.ps;
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Forward References.