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Bayesian Economic Cost Plans II. The Average Outgoing Quality

Abraham F. Jalbout [1*§], Hadi Y. Alkahby [2], Found N. Jalbout [3], Abdulla Darwish [3]

[1]Department of Chemistry, University of New Orleans, New Orleans, LA 70148-2820 USA, E-mail: Ajalbout@ejmaps.org
[2]Department of Mathematics, Dillard University, New Orleans, LA 70112 USA
[3]Department of Physics and Engineering, Dillard University, New Orleans, LA 70112 USA

*Author to whom correspondence should be addressed. §Speaker at the 16th Annual Conference on Applied Mathematics (CAM), University of Central Oklahoma, February 22-23, 2001

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Abstract: In recent years researchers in various quality control procedures consider the possibility of inspection errors as an important issue. The presence of these errors leads to changes in the so-called operational characteristic (O.C.) control curve, and as a result the average outgoing quality of an industrial process. We present a new mathematical model that can be applied to calculate such quantities as the expected number of defective items replaced in an accepted lot, and other functions of this process.

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1. Introduction

In attribute sampling plans the errors are generally of two kinds:

Type I error: a good item is classified as bad, with a probability e_1

Type II error: a bad item is classified as good, with a probability e_2

Collins and Case [1] derived an expression for the probability of acceptance under inspection errors. An expression was later derived for the marginal distribution of the observed defectives. (Hald [2-5] Case, Bennett and Schmidt [6-7] developed formulas for calculating the average outgoing quality (AOQ) when attribute inspection is subject to Type I and Type II inspection errors. Nine different rectification inspection policies are considered. These policies were first introduced by Wortham and Mogg [8]. Beainy and Case [9] later generalized these models, and they developed nine different sample/rest-of-lot disposition policies for single and double sampling along explicitly developed AOQ models. In this work both the attribute variable plans are considered [10] and a Bayesian technique is developed to estimate different parameters. Appendix A lists all of the notations used in this work. Although more the more recent work of Johnson, Kotz and Wu [11] describes some more modern industrial approaches to inspection errors with attributes in quality control, they still forget to include the Bayesian methods, a novel approach which is presented in this work.

2. Mathematical Development

Hald [2-5] has derived the following form of the marginal distribution of x:

$$g_n(x) = \binom{n}{x} p_x (1-p)^{n-x}, \quad x = 0,1,\cdots,n, \tag{2.1}$$

under the assumption that the number of defectives X in a lot size N is binomially

distributed, with a p.d.f:

$$f_N(X) = \binom{N}{X} p^X (1-p)^{N-X} \qquad X = 0,1,\cdots,N \tag{2.2}$$

where p is the process fraction defective.

The second assumption of equation (2.1) is that the number of defectives x in a sample size n given X is hypergeometric:

$$f(x \mid X) = \frac{\binom{n}{x}\binom{N-n}{X-x}}{\binom{N}{X}}, \tag{2.3}$$

thus this proves that the Hald's [2-5] derivations of the binomial distribution is reproduced by hypergeometric sampling. Thus for the Bayesian operating characteristic (BOC) curve the probability of lot acceptance is derived from the above equations as:

$$p_a = \sum_{x=0}^{c} g_n(x) = \sum_{x=0}^{c} \binom{n}{x} p^x (1-p)^{n-x} \quad x = 0,1,\cdots,n \quad 0 \le p \le 1 \tag{2.4}$$

where c is the acceptance number. For the inspection error analysis the observed defectives from a sample is replaced by observed number of defectives y_n. The probability of lot acceptance given in (2.4), will be reduced:

$$p_{ae} = \sum_{y_n=0}^{c} g_n(y_n) \tag{2.5}$$

where:

$$g_n(y_n) = \binom{n}{y_n} p_e^{y_n} (1-p_e)^{n-y_n}, \quad y_n = 0,1,\cdots,n, \tag{2.6}$$

Equation (2.4) gives the probability of lot acceptance for perfect inspection. The probability of lot acceptance when inspection errors are presented as p_{ae} in equation (2.5), and using equation (2.6) we can derive:

$$p_{ae} = \sum_{y_n=0}^{c} \binom{n}{y_n} p_e^{y_n} (1-p_e)^{n-y_n}. \tag{2.7}$$

We can now deduce an expression for the average outgoing quality (AOQ).

The AOQ can be defined as:

$$\begin{aligned} AOQ &= \frac{expected\ number\ of\ defective\ items\ remaining\ after\ inspection}{total\ number\ of\ items\ in\ the\ lot} \\ &= \frac{(N-n)pP_a}{N}. \end{aligned} \tag{2.8}$$

An expression fro AOQ can be derived by introducing the following terms: $p(N-M)$, the number of defectives in the uninspected portion of an accepted $p(N-M)e_2$, the number of defective of defective items classified as being good in the screened portion of the rejected lot. $n_p e_2$ is the number of defective items classified as good in the sample, $DITR$ is the number of defective items introduced through replacement into the lot. For an accepted lot, the expected number of defective items replaced in the lot is:

$$y = np_e. \tag{2.9}$$

The probability that an item is classified as being good is then:

$$P_g = (1-p)(1-e_1) + pe_2. \tag{2.10}$$

A set of n_1 items are selected at random, tested and classified as good or bad. A total of np_e items were needed to replace the defective items in the accepted lot. This procedure

of sampling defines a negative binomial process. The expected number of items tested to obtain np_a items, which are good, is then:

$$\frac{y}{P_E}. \tag{2.11}$$

The expected number of defective items replaced in an accepted lot is then:

$$DITR_a = Pe_2\left(\frac{y}{P_E}\right) \tag{2.12}$$

The expected number of defective items replaced in a rejected lot, which is screened, is:

$$DITR_r = pe_2\frac{(N-n)p_a}{P_E}. \tag{2.13}$$

The expected number of items to be replaced is:

$$\begin{aligned} DITR &= pe_2\left(\frac{y}{P_E}\right) + Pe_2\frac{(N-n)p_a}{P_E}(1-P_{aa}) \\ &= \frac{pe_2}{P_E}[y+(N-n)p_a(1-P_{aa})]. \end{aligned} \tag{2.14}$$

The expression AOQ is then:

$$AOQ = \frac{p(N-n)P_{aa} + p(N-n)(1-P_{aa})e_2 + npe_2 + DITR}{N}. \tag{2.15}$$

Expression (14) can be reduced to the form:

$$AOQ = \frac{npe_2 + p(N-n)(1-p_a)P_{aa} + p(N-n)(1-P_{aa})e_2}{N(1-p_a)}. \tag{2.16}$$

Similarly the AOQ expression for sampling with no replacement can be derived as:

$$AOQ = \frac{p(N-n)P_{aa} + p(N-n)(1-P_{aa})e_2 + npe_2}{N-np_a-(1-P_{aa})(N-n)p_a}. \tag{2.17}$$

This is true so no defectives are introduced through the replacement process.

3. Conclusions

In this work expressions for the average outgoing quality were derived for both the model involving replacement of defective items in the lot and when the items are not replaced. The potential application of this work lies in the ability of industrial researchers to calculate both of these quantities and decide the loss of such events, which are so very common in real life inspections. This simple model, is novel, and will be fruitful for any industrial process involving a constant inspection process. (see Jalbout, Alkabby [12])

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Appendix A: Notations Used in text

Notation	Definition
p	Fraction of items defective
p_e	Apparent fraction defective
e_1	Type I inspection error
e_2	Type II inspection error
N	Lot size
X	Value for measurable quality characteristic in variable sampling plans for fraction defective
x	Sample
n	Sample size
P_a	Probability of acceptance for a single variable sampling plan for fraction defective.
c	Acceptance number
i	Distribution of actual defectives
p_{ae}	Probability of acceptance when errors are present
y_e	Observed number of defectives
AOQ	Average outgoing quality
AOQ_e	Average outgoing quality error