A Selected Survey of Umbral Calculus*

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Submitted: April 28, 1995; Accepted: August 3, 1995

AMS Subject Classification. 05A40

1 What is the Umbral Calculus?

The theory of binomial enumeration is variously called the calculus of finite differences or the umbral calculus. This theory studies the analogies between various sequences of polynomials p_n and the powers sequence x^n. The subscript n in p_n was thought of as the shadow ("umbra" means "shadow" in Latin, whence the name umbral calculus) of the superscript n in x^n, and many parallels were discovered between such sequences.

Take the example of the lower factorial polynomials $(x)_n = x(x-1)\cdots(x-n+1)$. Just as x^n counts the number of functions from an n-element set to an x-element set, $(x)_n$ counts the number of injections. Just as the derivative maps x^n to nx^{n-1}, the forward difference operator maps $(x)_n$ to $n(x)_{n-1}$. Just as also polynomials can be expressed in terms of x^n via Taylor's theorem

$$f(x+a) = \sum_{n=0}^{\infty} \frac{a^n D^n f(x)}{n!},$$

Newton's theorem allows similar expressions for $(x)_n$

$$f(x+a) = \sum_{n=0}^{\infty} (a)_n \frac{\Delta^n f(x)}{n!}$$

*More up to date information may be available in the unofficial hypertext version of this survey at http://www.win.tue.nl/win/math/bs/statistics/bucchianico/hypersurvey.html .

†Author supported by NATO CRG 930554.

‡Author partially supported by EC grant CHRX-CT93-0400 and the "PRC maths-Info" and NATO CRG 930554.

where $\Delta f(x) = f(x+1) - f(x)$. Just as $(x+y)^n$ is expanded using the binomial theorem

$$(x+a)^n = \sum_{k=0}^{\infty} \binom{n}{k} a^k \, x^{n-k},$$

$(x+y)_n$ expands by Vandermonde's identity

$$(x+a)_n = \sum_{k=0}^{\infty} \binom{n}{k} (a)_k (x)_{n-k}.$$

And so on. [250, 233]

This theory is quite classical with its roots in the works of Barrow and Newton — expressed in the belief the some polynomial sequences such as $(x)_n$ really were just like the powers of x. Nevertheless, many doubts arose as to the correctness of such informal reasoning, despite various (see e.g., [42]) attempts to set it on an axiomatic base.

The contribution of Rota's school was to first set umbral calculus on a firm logical foundation by using operator methods [250, 340]. That being done, sequences of polynomials of binomial type

$$p_n(x+y) = \sum_{k=0}^{n} \binom{n}{k} p_k(x) p_{n-k}(y) \tag{1}$$

could be for once studied systematically rather than as a collection of isolated yet philosophically similar sequences. The sister sequence of divided powers $q_n(x) = p_n(x)/n!$ then obeys the identity

$$q_n(x+y) = \sum_{k=0}^{n} q_k(x) q_{n-k}(y). \tag{2}$$

Given any species of combinatorial structures (or quasi-species), let $p_n(x)$ be the number of functions from an n-element set to an x-element set enriched by this species. A function is enriched by associating a (weighted) structure with each of its fibers. All sequences of binomial type arise in this manner, and conversely, all such sequence are of binomial type.

2 History

As mentioned in the introduction, the history of the umbral calculus goes back to the 17th century. The rise of the umbral calculus, however, takes place in the second half of the 19th century with the work of such mathematicians as Sylvester (who invented the name), Cayley and Blissard (see e.g., [40]). Although widely used, the umbral calculus was nothing more than a set of "magic" rules of lowering and raising indices (see e.g., [165]). These rules worked well in practice, but lacked a proper foundation. Early attempts to put the umbral manipulation on an axiomatic basis (see [42]) were unsuccesful. Although the mathematical world remained sceptical of the umbral calculus, it was used extensively (e.g., in Riordan's highly respected book on combinatorics).

A second line in the history of the umbral calculus in the form that we know today, is the theory of Sheffer polynomials. The history of Sheffer polynomials goes back to 1880 when Appell studied sequences of polynomials $(p_n)_n$ satisfying $p_n' = n\, p_{n-1}$ (see [19]). These sequences are nowadays called Appell polynomials. Although this class was widely studied (see the bibliography in [119] which is included in the bibliography of this survey), it was not until 1939 that Sheffer noticed the similarities with which the introduction of this survey starts. These similarities led him to extend the class of Appell polynomials which he called polynomials of type zero (see [363]), but which nowadays are called Sheffer polynomials. This class already appeared in [242]. Although Sheffer uses operators to study his polynomials, his theory is mainly based on formal power series. In 1941 the Danish actuary Steffensen also published a theory of Sheffer polynomials based on formal power series [388]. Steffensen uses the name poweroids for Sheffer polynomials (see also [370, 369, 388, 389, 390, 387]). However, these theories were not adequate as they do not provide sufficient computational tools (expansion formulas etc.).

A third line in the history of the umbral calculus is the theory of abstract linear operators. This line goes back to the work of Pincherle starting in the 1890's (i.e., in the beginnings of functional analysis). His early work is laid down in the monumental monograph [291]. Pincherle went surprisingly far considering the state of functional analysis in those days, but his work lacked explicit examples. The same applies to papers by others in this field (see e.g., [116, 119, 429]).

A prelude to the merging of these three lines can be seen in [340], in which operators methods are used to free umbral calculus from its mystery. In [250] the ideas from [340] are extended to give a beautiful theory combining enriched functions, umbral methods and operator methods. However, only the subclass of polynomials of binomial type are treated in [250]. The extension to Sheffer polynomials is accomplished in [343]. The latter paper is much more geared towards special functions, while the former paper is a combinatorial paper.

The papers [250] and [343] were soon followed by papers that reacted directly on the new umbral calculus. E.g., Fillmore and Williamson showed that with equal ease the Rota umbral calculus could be situated in abstract vector spaces instead of the vector space of polynomials [139], Zeilberger noticed connections with Fourier analysis [443] and Garsia translated the operator methods of Rota back into formal power series [147].

We conclude this section with mentioning the remarkable papers [344, 127], in which the authors manage to make sense of the 19th century umbral calculus (thereby fulfulling Bell's dream [40]; cf. [309]).

3 Applications of the umbral calculus

We now indicate papers that apply the umbral calculus to various fields.

3.1 Combinatorics

An important property of (any extension of) the umbral calculus is that it has its own generalization of Lagrange's inversion formula (as follows from the closed forms for basic polynomials [250, Theorem 4]). Thus we find many papers in which new forms of the Lagrange's inversion formula is derived using umbral calculus [22, 33, 113, 176, 190, 191, 213, 263, 385, 417, 425].

In [227], the umbral calculus is generalized to symmetric functions. When counting enriched functions (functions, injections, reluctant functions, dispositions, etc.) from N to X, we can assign a weight to each function according to its fiber structure. $w(f) = \prod_{i\in N} f(i) = \prod_{x\in X} x^{|f^{-1}(x)|}$. The total number of such functions is a symmetric function $p_n(X)$ of degree n where $n = |N|$. The elementary and complete symmetric functions are (up to a multiple of $n!$) good examples of such sequences. They obey their own sort of binomial theorem

$$p_n(X \cup Y) = \sum_{k=0}^{n} \binom{n}{k} p_k(X) p_{n-k}(Y).$$

The generating functions of $p_n(X)$ are directly related to that of their underlying species. By specializing all x variables to 1, we return to the study to polynomials.

There are different ways of implementing q-analysis in terms of umbral calculus. The first q-umbral calculus can be found in [15]. Other q-umbral calculi can be found in [112, 114, 113, 182, 209, 322, 333]. Comparisons between different q-umbral calculi can be found in [9, 333]

Another rich field of application is linear recurrences and lattice path counting. Here we should first of all mention the work of Niederhausen [255, 254, 253, 257, 261, 262], but there other papers as well in this direction [316, 317, 338, 432, 433].

There are also papers on graph theory and umbral calculus, in particular on chromatic polynomials [310, 314].

General papers on the combinatorial properties of polynomials of binomial type are [166, 215, 214, 321, 397].

Coalgebraic aspects of umbral calculus are treated in [93, 95, 128, 207, 248, 251, 308].

3.2 Statistics

Non-parametric statistics (or distribution-free statistics) has a highly combinatorial flavor. In particular, lattice path counting techniques are often used. It is therefore not surprising that the main applications of umbral calculus to statistics are of a combinatorial nature [252, 256, 258, 262].

Another application concerns statistics for parameters in power series distributions [100, 220].

3.3 Probability theory

As suggested in [343, p. 752], there is a connection between polynomials of binomial type and compound Poisson processes. Two different approaches can be found in [91, 384].

A connection of polynomials of binomial type with renewal sequences can be found in [382].

Probabilistic aspects of Lagrange inversion and polynomials of binomial type can be found in [385].

Various probabilistic representations of Sheffer polynomials can be found in [125].

The following papers do not actually use umbral calculus, but provide interesting information on probabilistic aspects of generalized Appell polynomials [157, 158, 159, 236].

3.4 Topology

Applications of the umbral calculus to algebraic topology can be found in the work of Ray [308, 309, 312, 311, 313].

3.5 Analysis

Cholewinski developed a version of the umbral calculus for studying differential equations of Bessel type and related topics in [109].

A connection between approximation operators and polynomials of binomial type can be found in [187]. Further papers in this direction are [193, 245, 398]. Padé approximants are treated in [438].

Orthogonal polynomials play an important role in analysis. It is therefore important to know whether polynomials are orthogonal. The classification of orthogonal Sheffer polynomials was first found by Meixner [242]; it has been reproved many times (see e.g., [146, 205, 338, 363]).

General papers on orthogonal polynomials and umbral calculus are [152, 205, 307].

Hypergeometric and related functions are dealt with in an umbral calculus way in [409, 411].

There are different ways of implementing q-analysis in terms of umbral calculus. The first q-umbral calculus can be found in [15]. Other q-umbral calculi can be found in [10, 13, 112, 114, 113, 182, 209, 322, 333]. Comparisons between different q-umbral calculi can be found in [9, 333].

Constructing umbral calculi based on the operator $f(x) \mapsto \frac{f(x)-f(y)}{x-y}$ yields a powerful way to study interpolation theory [173, 327, 369, 418, 419, 420, 421].

Banach algebras are used by Di Bucchianico [123] to study the convergence properties of the generating function of polynomials of binomial type and by Grabiner [163, 164] to extend the umbral calculus to certain classes of entire functions.

Applications of umbral calculus to numerical analysis can be found in several papers of Wimp [437, 440, 441, 439].

3.6 Physics

An application of umbral calculus to the physics of gases can be found in [436].

Biedenharn and his co-workers use umbral techniques in group theory and quantum mechanics [47, 46].

Gzyl found connections between umbral calculus, the Hamiltonian approach in physics and quantum mechanics [167, 168, 169]. Closely related to this topic is the work by Feinsilver (see in particular [135, 134, 137]).

4 Generalizations of the umbral calculus

The umbral calculus of [343] is restricted to the class of Sheffer polynomials. It was therefore natural to extend the umbral calculus to larger classes of polynomials. Viskov first extended the umbral calculus to so-called generalized Appell polynomials (or Boas-Buck polynomials) [423] and then went on to generalize this to arbitrary polynomials [424]. The extension to generalized Appell polynomials makes it possible to apply umbral calculus to q-analysis (see section 3.5) or important classes of orthogonal polynomials like the Jacobi polynomials [333]. Roman remarks [332] that Ward back in 1936 attempted to construct an umbral calculus for generalized Appell polynomials [429]. Other interesting papers in this direction are [82, 126, 239].

An extension of the umbral calculus to certain classes of entire function can be found in [163, 164].

Another extension of the umbral calculus is to allow several variables [21, 68, 149, 193, 243, 278, 319, 324, 372, 430, 431]. However, all these extensions suffer from the same drawback, viz. they are basis dependent. Currently, an attempt is being undertaken by Rota, Loeb and Di Bucchianico to construct a basis-free umbral calculus in finite and infinite dimensions.

Roman [328, 329, 331, 332] developed a version to the umbral calculus for inverse formal power series of negative degree. Most theorems of umbral calculus have their analog in this context. In particular, any shift-invariant operator of degree one (delta operator) has a special sequence associated with it satisfying a type of binomial theorem. Nevertheless, despite its philosophical connections, this theory remained completely distinct from Rota's theory treating polynomials.

Later, in [233], a theory was discovered which generalized simultaneously Roman and Rota's umbral calculi by embedding them in a logarithmic algebra containing both positive and negative powers of x, and logarithms. A subsequent generalization [226, 228] extends this algebra to a field which includes not only x and $\log(x)$ but also the iterated logarithms, all of whom may be raised to any real power. Sequence of polynomials $p_n(x)$ are then replaced with sequence of asymptotic series p_a^α where the degree a is a real and the level α is a sequence of reals. Rota's theory is the restriction to level $\alpha = (0)$, and degree $a \in \mathbf{N}$. Roman's theory is the restriction to level $\alpha = (1)$ and degree $a \in \mathbf{Z}^-$. Thus, the difficulty in uniting Roman and Rota's theories was essentially that they lied on different levels of some larger yet unknown algebra. Other papers in this direction are [206, 265, 335, 336].

Rota's operator approach to the calculus of finite difference can be thought of as a systematic study of shift-invariant operators on the algebra of polynomials. The expansion theorem [250, Theorem 2] states that all shift-invariant operators can be written as formal power series in the derivative D. If $\theta : \mathbf{C}[x] \to \mathbf{C}[x]$ is a shift-invariant operator, then

$$\theta = \sum_{k=0}^{\infty} a_n D^n / n!$$

where $a_n = [\theta x^n]_{x=0}$.

However, a generalization of this by Kurbanov and Maksimov [219] to arbitrary linear operators has received surprisingly little attention. Any linear operator $\theta : \mathbf{C}[x] \to \mathbf{C}[x]$ can be expanded as a formal power series in X and D where X is the operator of multiplication by x. More generally, let B be any linear operator which reduces the degree of nonzero polynomials by one. (By convention, $\deg(0) = -1$.) Thus, B might be not only the derivative or any delta operator, but also the q-derivative, the divided difference operator, etc. Then θ can be expanded in terms of x and B:

$$\theta = \sum_{k=0}^{\infty} f_n(X) B^n .$$

Extensions of umbral calculus to symmetric functions have already been mentioned in section 3.1.

5 Further information

The bibliography of this survey are based on searches in the Mathematical Reviews and on the bibliographies in [343, 339] which have not yet been included completely. The bibliography contains papers on umbral calculus and related topcis such as Sheffer polynomials.

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