QUATERNION PROOF OF A THEOREM OF RECIPROCITY OF CURVES IN SPACE

By

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Quaternion Proof of a Theorem of Reciprocity of Curves in Space. By Sir William Rowan Hamilton, *LL.D., &c.*

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Let ϕ and ψ be any two vector functions of a scalar variable, and ϕ', ψ', ϕ'', ψ'' their derived functions, of the first and second orders. Then each of the two systems of equations, in which c is a scalar constant,

$$(1) \ldots \quad \mathrm{S}\phi\psi = c, \quad \mathrm{S}\phi'\psi = 0, \quad \mathrm{S}\phi''\psi = 0,$$

$$(2) \ldots \quad \mathrm{S}\psi\phi = c, \quad \mathrm{S}\psi'\phi = 0, \quad \mathrm{S}\psi''\phi = 0,$$

or each of the two vector expressions,

$$(3) \ldots \quad \psi = \frac{cV\phi'\phi''}{S\phi\phi'\phi''}, \quad (4) \ldots \quad \phi = \frac{cV\psi'\psi''}{S\psi\psi'\psi''},$$

includes the other.

If then, from any assumed origin, there be drawn lines to represent the reciprocals of the perpendiculars from that point on the osculating planes to a first curve of double curvature, those lines will terminate on a second curve, from which we can *return* to the first by a precisely similar process of construction.

And instead of thus taking the *reciprocal* of a *curve* with respect to a *sphere*, we may take it with respect to *any surface* of the *second order*, as is probably well known to geometers, although the author was lately led to perceive it for himself by the very simple *analysis* given above.