X-RAY CLUSTERS FROM A HIGH RESOLUTION HYDRODYNAMIC PPM SIMULATION OF THE CDM UNIVERSE

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ABSTRACT

A new three dimensional Piecewise Parabolic Method (PPM) based hydrodynamic code is utilized to compute the distribution of hot gas in the standard COBE normalized CDM universe. Utilizing periodic boundary conditions, a box with size $85h^{-1}$Mpc having cell size $0.31h^{-1}$Mpc, is followed in a simulation with $270^3 = 10^{7.3}$ cells. Adopting standard parameters determined from COBE and light element nucleosynthesis $\sigma_8 = 1.05$, $\Omega_b = 0.06$, we find the X-ray emitting clusters, compute the luminosity function at several wavelengths, the temperature distribution and estimated sizes as well as the evolution of these quantities with redshift. The results, which are compared with those obtained in the preceding paper (Kang *et al.*1994), may be used in conjunction with ROSAT and other observational data sets.

Overall, the results of the two computations are qualitatively very similar with regard to the trends of clusters properties, i.e., how the number density, radius and temperature depend on luminosity and redshift. The total luminosity from clusters is approximately a factor of two higher using the PPM code [as compared to the Total Variation Diminishing (TVD) code used in the previous paper] with the number of bright clusters higher by a similar factor.

The primary conclusions of the prior paper, with regard to the power spectrum of the primeval density perturbations are strengthened: the standard CDM model, normalized to the COBE microwave detection, predicts too many bright X-ray emitting clusters, by a factor probably in excess of five. The comparison between observations and theoretical predictions for the evolution of cluster properties, luminosity functions, size and temperature distributions, should provide an important discriminator among competing scenarios for the development

of structure in the universe.

Cosmology: large-scale structure of Universe – hydrodynamics – Radiation Mechanisms: Bremsstrahlung – X-ray: general

1. INTRODUCTION

The X-ray emission from hot gas in clusters of galaxies has been studied for over a generation (*cf* Gursky *et al.*1971), with new satellite observations by ROSAT (Trumper 1990) and other space instruments rapidly increasing our knowledge. It has recently been realized that these systems provide the opportunity for critical tests of cosmological theories proposing to explain the growth of structure in the universe (*cf* references in Frenk *et al.*1990 and in Ostriker 1993). The great clusters of galaxies, which have size of the order $1h^{-1}$Mpc and separations of order $50h^{-1}$Mpc, provide a measure of the nonlinearity of the density fluctuations on intermediate scales close to the usual normalization scale: $\sigma_8^2 \equiv \langle (\delta M/M)^2 \rangle_8$, where the subscript "8" designates the $8h^{-1}$Mpc scale, which is the radius of the top-hat filter used to calibrate the standard fluctuation measure.

In most optical studies of clusters one directly measures $\int n_g ds$ along the line of sight through the cluster or measures some moment of this such as $\int v_{||}^2 n_g ds / \int n_g ds$, where n_g is the galaxy volume density and $v_{||}$ the galaxy velocity dispersion. Since n_g is of order 10^3 times $\langle n_g \rangle$, and the cluster pathlength Δl is of order 10^{-3}Mpc times the Hubble length, c/H_0, the surface overdensities, $(\int n_g ds - \int \langle n_g \rangle ds) / \int \langle n_g \rangle ds$, are only of order unity, leading to intense discussion (*cf* Dekel *et al.*1989; Frenk *et al.*1990) concerning foreground and background contamination of samples, chance projection effects, etc. But the X-ray bremsstruhlung intensity towards a resolved cluster is proportional to $\int \rho^2 T^{-1/2} ds$, with the result that the contrast between the cluster and background in the X-ray band is enormous (of order $10^2 - 10^3$) in contrast to optical band measurements.

Theoretical predictions of cluster properties will of course be uncertain to the

extent that we do not know three things: $\Omega_b \equiv \rho_{baryon}/\rho_{crit}$; how large a fraction of the baryons has been transformed to galaxies; and finally how representative are the clusters, i.e., what is the ratio

$$b_x \equiv (\rho_{b,cl}/\rho_{tot,cl})/(\Omega_b/\Omega_{tot}). \quad (1)$$

On each of these issues our understanding has now reached the point where the uncertainties are minor. With regard to Ω_b, light element nucleosynthesis gives $\Omega_b = 0.06 \pm 0.01$ [we adopt a value slightly larger than that given by Walker *et al.*(1991), 0.05 ± 0.01] within the standard paradigm. In addition, since whatever this ratio is, it is not time dependent, we can make comparisons of the X-ray cluster luminosity functions (or other properties) at $z = 0$ with those at higher redshift independent of a precise knowledge of Ω_b. Second, recent observations have indicated that the baryons in the clusters are predominantly in the hot gaseous phase (White 1991), so corrections for the condensed (galactic) fraction are small. Finally, numerical hydrodynamical investigations such as the current and prior papers address this issue directly, indicating that the clusters are slightly anti-biased with respect to the global averages, i.e., the number $b_x \approx 1/1.5$.

In addition, the physical state of the gas in the great clusters is rather simple to model. In a cluster with central density $n_H = 1 \times 10^{-3}\mathrm{cm}^{-3}$ and central temperature $kT_x = 5\mathrm{keV}$ the cooling time is 2.0×10^{10} yrs, longer than the Hubble time. Thus, an adiabatic treatment is legitimate as long as the only entropy input is from shock fronts where atomic collisions transform the kinetic energy of converging gas flows into thermal energy.

The primary difficulty in accurately predicting the state of gas in the great

clusters is numerical. The ratio of cluster separation to cluster core size is greater than 10^2. Thus, in order to obtain both a reasonable statistical sample of clusters and also to resolve individual clusters, a ratio of box size to resolution element $(L/\Delta l) > 10^2$ is needed. This of course translates to $N > 10^6$ cells and dark matter particles in an Eulerian+PM code and requires that the true code resolution be of order the cell size or else a corresponding increase in machine capacity is required. ¿From initial exploratory work by Chiang, Ryu, & Vishniac (1989) and Ryu, Vishniac, & Chiang (1990) with 32^3 particles and cells to recent work (Cen & Ostriker 1993a,b) with 200^3 particles and cells there has been a steady improvement of the numerical treatment.

But these papers all used a diffusive (Richtmyer & Kinnard 1967) treatment of shocks with an "artificial viscosity" which spreads shocks over 4-5 cells. Recently 3-d shock capturing codes have been developed which contain shocks within two zones. Ryu *et al.*(1993) implemented the Total Variation Diminishing (TVD) scheme for cosmological applications and the preceding paper (Kang *et al.*1994 = KCOR) used this code to study the standard CDM scenario.

Needless to say, serious questions remain concerning the accuracy and reliability of the numerical codes. We are currently at work on a detailed comparison of several different codes, including Eulerian as well as Smoothed Particle Hydrodynamic (SPH) methods (Kang *et al.*1994). The SPH (Lagrangian) approach taken by Evrard, Davis, & Summers (1994) and Katz, Hernquist & Weinberg (1993) has considerably higher spatial resolution than do equivalent Eulerian methods, but this is obtained at a cost that makes investigation of large cosmological volumes possible only with relatively poor mass resolution. Here we use an alternative Eulerian code based on the Piecewise Parabolic Method (PPM),

which should have resolution higher than that of the TVD code utilized in the prior paper. The accuracy of energy conservation is, however, slightly lower in this PPM code.

The two codes have been developed independently and a comparison of the results obtained by the two methods utilizing identical initial conditions should help indicate which results are robust and which are still significantly uncertain due to numerical effects. In §2 we outline the method and initial conditions, in §3 the results and in §4 we assemble our conclusions with primary emphasis on the comparison between the PPM and TVD results.

2. METHOD AND INITIAL CONDITIONS

2.1 Method

A hybrid code was used to perform these calculations; a standard N-body particle code (more specifically, a particle-mesh technique) was combined with our implementation of the grid-based piecewise parabolic method (PPM) to solve the hydrodynamic equations. The PPM algorithm is described by Colella and Woodward (1984) and provides higher order accuracy along with the ability to capture shocks in a small number of zones while avoiding post shock oscillations. The net result is an increase in resolution for the same number of zones, crucial for cosmology, where many length scales are important.

However, a number of modifications were required to make the method suitable for cosmological simulations. These modifications, along with the details of testing, described in Bryan *et al.*(1993), are briefly mentioned here. Gravitational forces both from the gas and the dark matter were included, as was the Hubble expansion. The code solves both the total energy and thermal energy

equations in order to obtain the correct pressure distribution, both in cold flows, where kinetic energy terms dominate, and in strong shocks, where most of the energy is thermalized. Finally, a new pressure interpolation was adopted for thin, collapsing structures in order to better model strongly peaked features.

The PPM technique has been subjected to many hydrodynamic tests, some of which are described in Woodward and Colella (1984). We have also applied this specific code to a suite of cosmological test problems, including a one dimensional Zeldovich pancake collapse and resolution studies involving the standard CDM model.

2.2 Initial Conditions

We adopt the standard CDM power spectrum with the transfer function given by Bardeen *et al.*(1984). The following parameters are used: $n = 1$, $h = 0.5$, $\Omega = 1$, $\Omega_b = 0.06$ and $\sigma_8 = 1.05$, the same as utilized by KCOR. Note that the amplitude normalization of the power spectrum is determined by COBE observations (*cf* Efstathiou, Bond & White 1992), parameterized by σ_8 to translate into conventional notation. Our box size is $85h^{-1}$Mpc with 270^3 cells and 135^3 dark matter particles (again the same as KCOR) so our nominal resolution is $0.31h^{-1}$Mpc. The choice of Ω_b is consistent with light element nucleosynthesis (Walker *et al.*1991). The same initial conditions were used as in KCOR.

3. RESULTS

The X-ray clusters in the simulation are identified as follows. We first calculate the total X-ray luminosity due to thermal Bremsstrahlung (assuming privemal composition and neglecting lines) for each cell given the cell density and

temperature assuming cosmic abundances with both hydrogen and helium fully ionized. The detailed formulae were presented in KCOR. Note that in the following discussion all units of length are given in co-moving not metric coordinates.

The cells with the total X-ray luminosity higher than 10^{38}erg s^{-1} are selected as X-ray bright cells. Then we find the local maxima (by comparing L_{ff} of each X-ray bright cell with that of 26 neighboring cells) and identify them as the centers of the X-ray clusters. Having defined the centers of the X-ray clusters, we go back to the whole simulation box to define our X-ray clusters. We analyze the simulation in two different ways (which correspond to spheres of radius $0.5h^{-1}Mpc$ and $1.0h^{-1}$Mpc) as follows. First, each cluster core consists 27 cells (26 cells surrounding the central cell plus the central cell). These 27 cells are weighted so that total volume of the cluster equals the volume of a sphere of radius $0.5h^{-1}Mpc$ as appropriate for observationally defined X-ray clusters. That is, we define a weight $w = R_v$ for the central cell and the 6 cells that share the faces with the central cell, $w = (3/4)R_v$ for 12 cells that share the edges with the central cell, and $w = (1/2)R_v$ for 8 cells that shares only the corners with the central cell, where $R_v = (4\pi\ 0.5^3/3)/(20\Delta x^3) = 0.839$. This weighting scheme compensates for the adoption of slightly too large a volume by lowering the weight per cell. For concentrated clusters we may be underestimating the luminosity by as much as 16% a correction small compared to other errors. Second, the total X-ray cluster volume consists 125 cells (124 cells surrounding the central cell plus the central cell) and each cell is weighted equally which gives a volume equivalent to a sphere of radius $0.98h^{-1}$Mpc.

Our $(85h^{-1}\text{Mpc})^3$ box at $z = 0$ contained (1,0) clusters with (total, core) luminosity brighter than 10^{45}erg/s, (10,4) brighter than 10^{44}erg/s, (43,29) brighter

than 10^{43}erg/s, (189,127) brighter than 10^{42}erg/s and (433,360) brighter than 10^{41}erg/s. We attribute the small number of clusters brighter than 10^{45}erg/s as simply due to the size of our box. An additional factor of at least two in scale (and 8 in computer resources) would be required to significantly improve on the quoted results. An immediate comparison to the KCOR results is possible if one notes that for $L_x > 10^{44}$erg/s that paper found (8,1) clusters with (total, core) luminosity higher than the specified limit compared to (10,4) in this work. This paper finds higher luminosities, especially core luminosities, due to slightly higher resolution. But the difference is only a bit larger than would be expected due to Poisson statistics were we to repeat the simulation with the same power spectrum but a different choice of random numbers.

It is convenient to fit the luminosity function to the three parameter Schechter function:

$$n(L)dL = n_0(L/L_*)^{-\alpha} e^{-L/L^*} d(L/L^*). \tag{2}$$

Luminosity functions for cluster cores were computed in four frequency bands: total Bremsstrahlung luminosity, $0.3-3.5$keV, $0.5-4.5$keV and $2-10$keV and are displayed in Figures 1a, 2a, 3a and 4a, respectively. The results for entire clusters (emission from a $1h^{-1}$Mpc sphere) are presented for the same frequency bands in Figures 1b, 2b, 3b and 4b. The figures show the range of cluster properties which are most accessible to observation: 10^{40}erg/s$\leq L_x \leq 10^{44}$erg/s and $0 \leq z \leq 1$. Also shown in Figures (1b) and (4b) are the observations from Henry & Arnaud (1991) and from Henry (1992), respectively. We see that the computed number densities of bright clusters ($h^2 L \sim 10^{43}$erg/s) are above the observed ones by a factor of 5 to 10. We have computed approximate Schechter

function fits to the results, with the numerical parameters $(n_0\,, L^*, \alpha\,)$ as a function of redshift collected in Table (1), and the simulated data extended to $z = 5$. Also in Table (1) we integrate over the cluster luminosity function, using the Schechter fit $j_{cl} \equiv n_0 L^* \Gamma(2-\alpha)$, showing the result in the second to last column and give also, in the last column, the total emissivity from the box as j_{gas}, which includes the emission from lower density regions further from cluster cores than $0.5h^{-1}$Mpc and also from clusters whose central emissivity is less than our cutoff value. We estimate that α is well constrained (± 0.03) and the product $n_0 L^*$ is also fairly well constrained, but individual values (n_0, L^*) are poorly determined because L^* is dependent on the quite uncertain highest luminosity clusters. To estimate the purely statistical uncertainty we reanalyzed the $z = 0$ data from the lower panel of Table 1a (total, integrated X-ray cluster properties) looking separately at two halves of the box. The fractional differences $(\Delta Q\,/Q\,)$ for $Q \equiv (\alpha\,, L_x^*, kT_x, n_0\,, j_{cl}\,, j_{gas}\,)$ were found to be $(0.032, 0.83, 0.20, 0.54, 0.38, 0.33)$ respectively. The fact that even the integral j varies significantly between the two halves of the box reminds us again of the "cosmic variance". Our sample volume is not large enough to give us a robust estimate for the cosmic mean value of j.

We see that the cluster cores, as we have defined them, contain between 1/2 and 1/3 of the total X-ray emission in the regions studied, comparable to the results of KCOR. The total cluster luminosity in the box is typically 5/6 of the X-ray emission from the box, a slightly higher fraction than found by KCOR although the bright clusters ($L \geq 10^{43}$erg/s) occupy a volume which is less than 10^{-3} of the total, i.e., the emissivity is very concentrated at bright peaks. For the total luminosity the Schechter α parameter is approximately 1.4 (as compared

to 1.5 in KCOR) with little evolution and α for the few keV bands is typically slightly flatter at 1.2 (compared to 1.4 in KCOR). For $\alpha < 2$, as we have noted, most of the luminosity arises from bright clusters. These differences between the two methods are of marginal statistical significance.

The number density of bright clusters peaks at intermediate redshift and the typical luminosity is (for small redshift) relatively constant, so there is a peak emissivity at approximately $z = 0.5-1.0$ for the several keV bands. Thus, crudely speaking, in this model one expects weak "positive" evolution until nearly $z = 1$ and then negative evolution thereafter.

We believe that the peak in emission seen in both simulations at moderate redshift is real. The reasons for this are discussed in KCOR. The approximate Press-Schechter formalism, which does not allow for a variety of effects, can not easily mimic the full non-equilibrium hydrodynamic treatment.

While the integrated X-ray emissivity evolves fairly slowly over the period surveyed in Table (1a), L_* and n_0 tend to evolve more rapidly and in opposite directions, with more and more lower and lower luminosity clusters at higher redshifts. To highlight the negative evolution of the bright end of the luminosity function we computed the fifth and sixth columns of Tables 1b-1d (sixth and seventh columns of Table 1a), the comoving density of clusters having luminosity greater than 10^{43}erg/s and 10^{44}erg/s. For reasons stated earlier (based on our limited box size) we use the Schechter fit rather than direct counts to compute these columns. Comparing columns 4 and 6 (5 and 7 of Table 1a) we see that, although the total number density n_0 of clusters increases with redshift (until $z = 3$), the number density of the highest luminosity ($L_x > 10^{44}$erg/s) clusters decreases for $z > 0.5$ (the same place as the peak was found in KCOR). This is

presumably one of the effects leading to the observational appearance of "negative evolution". Statistical fluctuations in our results are still quite significant due to the limited box size, especially for $n(L_x > 10^{44}\mathrm{erg/s})$.

Redshift effects strongly exaggerate this tendency to observe negative evolution, since higher redshift clusters tended to have lower temperatures (*cf* column 4 of Table 1a and Figures 5, 12), and both effects will reduce the energy observed by satellites measuring the X-ray flux in high energy bands. Note that the negative evolution in the density of clusters with $L > 10^{44}\mathrm{erg/s}$ is more and more steep in Tables (a$\rightarrow$ d) as one looks at higher energy bands, although statistical fluctuations are large.

The emission-weighted temperature, T_x of each cluster is calculated and the distributions are shown in Figure (5). The arrow in each panel indicates the average cluster temperature (weighted by luminosity) at the given epoch. Also shown in Figure (5b) (corss sahded area) is the observed temperature function from Henry & Arnaud (1991) as the cross-shaded area. Note that the computed temperature function is higher than the observed one and shows a much flatter slope than its observational counterpart at the high temperature end. This is consistent with other, independent tests such as the cluster mass function (Bahcall & Cen 1992). We see that at all epochs the coolest clusters dominate the statistics (the turnover at low T_x is presumably caused by our definition of minimun cell luminosity to constitute an X-ray bright cell), but the mean is determined by the high mass, high luminosity, high temperature end of the distribution. The increase of the mean with increasing time is for the usual reason, $T_x \propto (H\lambda)^2$, with the wavelength of nonlinear waves increasing with time faster than H decreases. The mean temperatures, indicated by arrows, are included

in column 3 of Table (1a). The results are very close to those found in KCOR with $(\bar{T}_{PPM}/\bar{T}_{TVD}) = 0.99$ for entire clusters after averaging over the five most recent tabulated cosmological epochs. Looking ahead to Figure (7a) we see the strong correlation found between T_x and total luminosity (clusters are shown at $z = 0$).

Now let us turn again to the total cluster luminosity (vs core luminosity) as shown in the lower panels of the tables and in figures 1b-7b, the quantity normally measured by satellite observations. The ratio of $(j_{cl,tot}/j_{cl,core})$ is near 2.0 ± 0.1 for the redshift range $0 < z < 1$, slightly, and perhaps not significantly, less than the ratio 2.2 ± 0.1 found by KCOR.

We can also roughly estimate the effective radii of the clusters by assuming that the emission has a profile

$$j = \frac{j_0}{[1 + (r/r_x)^2]^2} \tag{3}$$

and determining, from the ratio of the luminosity (integrated over frequency) of the central cell to the total cluster luminosity, the value of r_x which would produce this ratio. We show in Figure (6) the radii determined in this fashion. The peaks seen in the panels of Figure (6) are, of course, artificially induced by our cell size of $0.31h^{-1}$Mpc, but the distribution to larger radii should be reasonably accurate. Arrows indicate the luminosity-weighted average values. Since brighter clusters tend to be resolved, these numbers should be reliable. We see a weak trend of increasing size with increasing time which is in the theoretically anticipated direction. Longer wavelengths became nonlinear later producing larger clusters, and smaller clusters merge to produce larger clusters with increasing time.

Now in Figures (7a,b) we show the scatter plots of $(T_X\,, r_x)$ vs L_{tot} (integrated over frequency). We see that there is a clear correlation between L and T_x. But we do not see any strong correlation between L and r_x. The best fit lines (dashed) indicate that a slope of (0.42,0.34) for core and total cluster temperature. In KCOR we found for these slopes (0.39,0.37) respectively. The observed correlation [*cf* Figure (7a) shown as cross-shaded area] between $\bar{T}_x$ and total cluster luminosity is $\log_{10}\bar{T}_x(\mathrm{keV}) = \log_{10}(4.2^{+1.0}_{-0.8}) + (0.265 \pm 0.035)\log_{10}(h^2 L_{44})$ according to Henry & Arnaud (1991).

Finally, we address temperature variations within clusters. Given our limited resolution there is little that can be accurately determined on this issue from our simulations, but we are able to compare the central cell (Volume= $3.1 \times 10^{-2} h^{-1}\mathrm{Mpc}^3$) with the surrounding cells ($3^3 - 1^3$) (volume of size $5.9 \times 10^{-1} h^{-3}\mathrm{Mpc}^3$) and the cells surrounding these cells ($5^3 - 3^3$, vol= $3.0 h^{-3}\mathrm{Mpc}^3$). We define the ratio of the inner cell to the next cube as T_c/T_{shell} (volume-weighted average) and show the scatter diagram in Figure (8). No trend is seen with luminosity and the median value, indicated by the dashed line is 1.5. The cluster gas deviates significantly from isothermality with a $5\% - 10\%$ temperature decline typically found by a radius of $0.4 - 0.5 h^{-1}\mathrm{Mpc}$ but a sharp fall off indicated by $1 h^{-1}\mathrm{Mpc}$. In Figure (9) we compare the (luminosity-weighted) temperatures found in the three regions noted above, normalized to the temperature in the central cell. The large dispersion is indicated by the error bar. Plate A shows the temperature and density contours through a slice of size $85 \times 85 \times 0.31 h^{-1}\mathrm{Mpc}$. A slight temperature minimum is seen in the center of several of the clusters. The physical explanation for this is given in KCOR, and the high resolution of the PPM code allows the effect to be seen directly.

4. CONCLUSIONS

In Figures (1-4) we showed the evolution of the cluster luminosity function expected in the CDM scenario. In the easily observed range of parameters ($0 \leq z \leq 1$, 10^{40}erg/s$\leq L_x \leq 10^{44}$erg/s) little evolution is seen (for comoving observers) in any of the computed bands aside from a decline in the number of brightest sources (somewhat uncertain due to our limited box size) and a modest increase (by about a factor of two) in the luminosity function for fainter objects. The likely explanation for this has been mentioned: it is largely coincidental and due to the balancing of two effects: new breaking waves increase the luminosity density but mergers decrease it. It just happens that for this spectrum at this epoch the net rate of change is small. in other scenarios we expect the results to be different, but only detailed calculations can prove this.

However, Figures 5 and 6 did show substantial rates of change in other quantities, the temperatures and clusters radii. These important trends are summarized in Figures 10 and 11 where we see a factor of 2-3 decline in both these quantities by redshift 1 and a 37% decline in the (luminosity-weighted) mean temperature even in the small redshift range $z = 0 \rightarrow 0.5$. This trend should be detectable even with a relatively "soft" X-ray instrument such as ROSAT. It provides a test of $\Omega = 1$ models since in open, $\Omega < 1$ models the structure is expected to freeze out at early epochs ($z \sim 1/\Omega$) with cluster temperatures not changing substantially after that time. In Figures (10) and (11) we also show, as solid lines, the best fit power law dependences $R_x \propto (1+z)^{-2}$ and $T_x \propto (1+z)^{-1}$ anticipated by Kaiser (1986). We see that the temperature trend is as expected but the variation in radius is less steep than anticipated by the simple theoretical modelling, although of course our accuracy for R_x is affected by our limited

resolution.

The primary conclusion of KCOR was that this model for growth of structure (standard CDM as normalized to COBE) produces too much structure at intermediate scales. Specifically there are too many bright X-ray clusters produced (*cf* Figure 4b) and too much integrated background X-ray intensity. Those results are strengthened by the current paper which confirms all the trends seen in the earlier paper but finds still more high luminosity clusters, further increasing the discrepancy with observations.

At fixed luminosity the predicted number density is an order of magnitude to high and at a fixed number density the luminosity is perhaps a factor of three too large. Thus, to fit observations we would need to reduce the density Ω_b by more than 40% which would put it more than one sigma below the Walker *et al.*(1991) value. But, even if we were to fit the observed luminosity function by this contrivance, the model would still fail due to the fact that the predicted temperatures are too high [*cf* Figures (5b) and (7b)]. Improvement of our numerical techniques to obtain higher spatial resolution would lead to a further increase in these discrepancies. Frenk *et al.*(1990) combined hydrodynamical modelling of individual clusters with large scale dark matter P^3M simulations. They found that CDM with an amplitude of $\sigma_8 = 0.4 - 0.5$ would produce a satisfactory fit to observations but that unbiased models (such as that adopted here to match the COBE normalization) would produce too many high luminosity, high temperature clusters. The direct methods adopted in KCOR and the present paper confirm and strengthen these earlier results.

The methods used here, if combined with adaptive mesh refinement, should enable one to compare details of cluster structure with developing X-ray observa-

tions. Comparing results from simulations based on other cosmological scenarios for the growth of structure should help us decide which, if any, of our current models is correct.

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FIGURE CAPTION

Fig. 1– Figure (1a): the X-ray cluster bremsstrahlung luminosity (from central $< 0.5h^{-1}Mpc$ regions) function integrated over the whole frequency range at five different redshifts $z = (0, 0.2, 0.5, 0.7, 1.0)$. Figure (1b): the X-ray cluster bremsstrahlung luminosity (from $< 1.0h^{-1}$Mpc region) function integrated over the whole frequency range (filled dots) at same five different redshifts. The cross-shaded area shows the observations (Henry & Arnaud 1991, $\{3.1^{+4.5}_{-1.8} \times 10^{-6} h^3 \mathrm{Mpc}^{-3} h^2 [L_{44}(\mathrm{bol})]^{-1}\} \times [h^2 L_{44}(\mathrm{bol})]^{-1.85 \pm 0.4}$).

Fig. 2– Same as Figure (1) but for the luminosities integrated over $0.3-3.5$keV frequency bin.

Fig. 3– Same as Figure (1) but for the luminosities integrated over $0.5-4.5$keV frequency bin.

Fig. 4– Same as Figure (1) but for the luminosities integrated over $2-10$keV frequency bin. The cross-shaded area in (4b) indicates the observation (Henry 1992).

Fig. 5– The X-ray cluster temperature (T_x, emission-weighted temperature) function at six different redshifts $z = (0, 0.5, 1, 2, 3, 5)$. Arrows indicate the luminosity-weighted average temperature $\bar{T}_x$ at each epoch. In the first ($z = 0$) panel in (5b) the cross-shaded area is the observed temperature function from Henry & Arnaud (1991) $[(1.8^{+0.8}_{-0.5} \times 10^{-3} h^3 \mathrm{Mpc}^{-3} keV^{-1})(kT)^{-4.7 \pm 0.5}]$.

Fig. 6– The X-ray cluster effective radius (r_x) distribution [cf equation (2)]. Arrows indicate the luminosity-weighted effective radius at each epoch.

Fig. 7– Figure (7a) shows the scatter plot of T_x vs L_{tot} at $z = 0$. The cross-shaded area line indicates the observations of Henry & Arnaud (1991). The dashed line is the best fit of the simulation results. Figure (7b) shows the scatter plot of r_x vs

L_{tot} at $z = 0$.

Fig. 8– The ratio of the central cell temperature to the temperature of its surrounding shell ($\sim$ one cell thick) as a function of L_{tot}.

Fig. 9– We compare the (luminosity-weighted) temperatures found in the three regions (central cell, the shell surrounding the central and the next outer shell) and normalized to the temperature in the central cell. Note the errorbars are 1σ.

Fig. 10– The average cluster core radii in physical units as a function of redshift for clusters with luminosity in the $0.5 - 4.5$keV band greater than 10^{43}erg/s.

Fig. 11– The average cluster temperature as a function of redshift for clusters with luminosity in the $0.5 - 4.5$keV band greater than 10^{43}erg/s.

Table 1a. Parameters of Schechter fits for the X-ray cluster luminosity function integrated over the entire frequency range.

X-ray Cluster Core Luminosity ($< 0.5h^{-1}$Mpc)

z	α	L_x^* (10^{44})	$k\bar{T}_x$ (keV)	n_0	$n(L > 10^{43})$	$n(L > 10^{44})$	j_{cl}	j_{gas}
0	1.44	5.82	7.63	3.88	17.1	3.32	0.36	0.87
0.2	1.35	1.16	4.09	18.6	25.0	2.02	0.30	0.76
0.5	1.51	4.80	4.44	4.61	21.8	3.48	0.40	0.95
0.7	1.39	0.94	3.09	26.0	31.7	1.86	0.36	0.85
1	1.38	0.84	2.49	32.1	35.2	1.81	0.39	0.84
2	1.44	0.59	1.02	33.3	28.9	0.75	0.31	0.63
3	1.32	0.17	0.89	81.7	16.4	8.0×10^{-3}	0.18	0.36
5	1.57	0.20	0.47	5.62	1.46	1.3×10^{-3}	0.02	0.04

X-ray Cluster Total Luminosity ($< 1h^{-1}$Mpc)

z	α	L_x^* (10^{44})	$k\bar{T}_x$ (keV)	n_0	$n(L > 10^{43})$	$n(L > 10^{44})$	j_{cl}	j_{gas}
0	1.40	11.2	5.80	4.50	25.2	6.32	0.74	0.87
0.2	1.31	2.39	3.51	19.8	39.2	6.19	0.62	0.76
0.5	1.45	9.20	3.62	5.48	32.3	7.07	0.81	0.95
0.7	1.34	1.86	2.53	27.8	50.1	6.33	0.71	0.85
1	1.29	1.01	2.04	52.0	58.5	4.47	0.67	0.84
2	1.31	0.56	1.35	64.2	47.1	1.40	0.47	0.63
3	1.18	0.18	0.86	138.6	30.0	0.026	0.29	0.36
5	1.50	0.20	0.48	8.07	2.07	1.7×10^{-3}	0.03	0.04

Here L_x^* and L are in units of 10^{44}erg/s; n_0, $n(L > 10^{43})$ and $n(L > 10^{44})$ are in units of $10^{-6}h^3$Mpc^{-3}; j_{cl} and j_{gas} are in units of 10^{40}erg/s/h^{-3}Mpc^3, and j_{cl} may be larger than j_{gas} due to the inaccuracy of the Schechter fit.

Table 1b. Parameters of Schechter fits for the X-ray cluster luminosity function in $0.3 - 3.5$keV band

X-ray Cluster Core Luminosity ($< 0.5h^{-1}$Mpc)

z	α	L_x^*	n_0	$n(L > 10^{43})$	$n(L > 10^{44})$	j_{cl}	j_{gas}
0	1.33	0.63	13.0	10.7	0.38	0.11	0.33
0.2	1.20	0.27	45.3	15.7	8.0×10^{-2}	0.14	0.37
0.5	1.32	0.48	26.2	17.6	0.36	0.17	0.45
0.7	1.24	0.27	57.4	20.2	9.5×10^{-2}	0.19	0.45
1	1.22	0.23	76.6	22.4	5.8×10^{-2}	0.21	0.46
2	1.29	0.22	62.5	17.6	3.3×10^{-2}	0.18	0.35
3	1.31	0.089	78.3	5.20	1.5×10^{-5}	0.091	0.19
5	1.65	0.19	2.92	0.71	3.3×10^{-4}	0.014	0.014

X-ray Cluster Total Luminosity ($< 1h^{-1}$Mpc)

z	α	L_x^*	n_0	$n(L > 10^{43})$	$n(L > 10^{44})$	j_{cl}	j_{gas}
0	1.27	1.26	15.8	20.0	2.02	0.25	0.33
0.2	1.14	0.44	55.3	29.2	0.70	0.27	0.37
0.5	1.26	0.98	29.1	30.9	2.39	0.35	0.45
0.7	1.21	0.62	51.3	37.4	1.60	0.37	0.45
1	1.17	0.42	80.9	42.0	0.85	0.38	0.46
2	1.26	0.41	56.7	30.5	0.50	0.29	0.35
3	1.20	0.17	70.1	14.1	8.6×10^{-3}	0.14	0.19
5	1.44	1.43	1.24	2.22	0.19	0.028	0.014

Table 1c. Parameters of Schechter fits for the X-ray cluster luminosity function in $0.5 - 4.5$keV band

X-ray Cluster Core Luminosity ($< 0.5h^{-1}$Mpc)

z	α	L_x^*	n_0	$n(L > 10^{43})$	$n(L > 10^{44})$	j_{cl}	j_{gas}
0	1.30	0.62	13.5	10.7	0.39	0.11	0.34
0.2	1.15	0.21	52.7	13.7	2.7×10^{-2}	0.12	0.37
0.5	1.29	0.48	26.3	16.6	0.37	0.16	0.44
0.7	1.24	0.34	46.4	20.7	0.21	0.19	0.43
1	1.22	0.32	56.9	23.7	0.21	0.22	0.44
2	1.31	0.29	40.9	16.0	8.5×10^{-2}	0.16	0.30
3	1.31	0.10	60.1	5.03	5.2×10^{-5}	0.079	0.15
5	1.65	0.09	1.60	0.09	1.7×10^{-7}	0.004	0.009

X-ray Cluster Total Luminosity ($< 1h^{-1}$Mpc)

z	α	L_x^*	n_0	$n(L > 10^{43})$	$n(L > 10^{44})$	j_{cl}	j_{gas}
0	1.23	1.36	17.2	21.5	2.50	0.28	0.34
0.2	1.14	0.47	48.7	27.2	0.75	0.25	0.37
0.5	1.24	0.99	28.3	29.5	2.39	0.34	0.44
0.7	1.19	0.66	47.7	35.7	1.76	0.36	0.43
1	1.16	0.42	71.8	37.1	0.76	0.34	0.44
2	1.28	0.45	42.2	25.0	0.49	0.24	0.30
3	1.21	0.18	50.0	10.8	8.9×10^{-3}	0.11	0.15
5	1.43	5.95	0.47	2.05	0.41	0.044	0.009

Table 1d. Parameters of Schechter fits for the X-ray cluster luminosity function in $2-10$keV band

X-ray Cluster Core Luminosity ($< 0.5h^{-1}$Mpc)

z	α	L_x^*	n_0	$n(L>10^{43})$	$n(L>10^{44})$	j_{cl}	j_{gas}
0	1.21	0.74	13.1	10.8	0.62	0.11	0.35
0.2	1.20	0.62	18.9	13.6	0.59	0.14	0.31
0.5	1.31	0.90	11.0	11.7	0.75	0.13	0.35
0.7	1.25	0.51	21.2	13.7	0.38	0.13	0.30
1	1.24	0.59	20.6	14.8	0.55	0.15	0.27
2	1.38	0.53	9.35	6.95	0.16	0.07	0.14
3	1.30	0.07	20.0	0.78	1.6×10^{-7}	0.018	0.038
5	1.33	0.40	0.51	0.28	3.8×10^{-3}	2.8×10^{-3}	1.4×10^{-3}

X-ray Cluster Total Luminosity ($< 1h^{-1}$Mpc)

z	α	L_x^*	n_0	$n(L>10^{43})$	$n(L>10^{44})$	j_{cl}	j_{gas}
0	1.22	2.46	9.49	16.1	3.00	0.28	0.35
0.2	1.18	1.18	18.8	20.2	2.21	0.25	0.31
0.5	1.24	1.36	15.4	19.6	2.23	0.25	0.35
0.7	1.23	1.12	19.1	21.3	2.02	0.26	0.30
1	1.21	0.91	24.4	23.2	1.78	0.26	0.27
2	1.31	0.69	12.5	10.9	0.47	0.11	0.14
3	1.26	0.15	16.0	2.68	7.0×10^{-4}	0.030	0.038
5	1.62	0.20	0.18	0.05	3.0×10^{-5}	8.2×10^{-4}	1.4×10^{-3}