Nonlinear Growth of Dynamical Overstabilities in Blast Waves

Mordecai-Mark Mac Low[1,2]
Space Science Division, Mail Stop 245-3, NASA Ames Research Center, Moffett Field, CA 94035-1000

Michael L. Norman[2]
National Center for Supercomputing Applications, 5600 Beckman Institute, Drawer 25, 405 North Mathews Avenue, Urbana, IL 61801

Astrophysical Journal, 1993, **407**, 207

Abstract

Blast waves have been shown by Vishniac to be subject to an oscillating instability, with growth occurring when the blast wave is thin enough. The nonlinear development of this overstability has not yet been determined. We use the numerical gas dynamics code ZEUS-2D to directly model the overstability. By perturbing a blast wave with a low amplitude eigenfunction of the overstability we confirm the linear analysis. We then increase the amplitude of the perturbations to determine the nonlinear behavior of the overstability. We find that the overstability saturates due to weak transverse shocks in the shell. Transverse velocities in the dense shell reach the post-shock sound speed, and high density regions with sizes of order the shell thickness form. Transverse oscillations continue even after saturation. This confirms and explains the damping of the overstability experimentally discovered by Grun et al. Our results support the short time scale for gravitational instability of the shell proposed by Vishniac. Observations of thin, radiatively cooled shells in supernova remnants may directly show the effects of the overstability.

Subject headings: Hydrodynamics—Nebulae: Supernova Remnants—Shock Waves

1 Introduction

Until recently, blast waves in homogeneous gases were thought to be stable (*e.g.* Erpenbeck 1962). However, a linear perturbation analysis by Vishniac (1983) showed that blast waves in gas with high compressibility (adiabatic index γ approaching unity) are subject to growing overstable oscillations. This analysis was confirmed by Bertschinger (1986), and the eigenmodes of the overstability were found by Ryu & Vishniac (1987). As a rule of thumb,

---

[1]Also Department of Astronomy, University of California at Berkeley

[2]Also Department of Astronomy, University of Illinois at Urbana-Champaign

blast waves from point explosions are overstable if the density ratio across the shock exceeds ten, and blast waves from bubbles with continuous energy input are overstable with density ratios exceeding twenty-five (Ryu & Vishniac 1987, 1988). Such conditions occur in radiative shocks in the interstellar medium and elsewhere.

The overstability acts in at least two astrophysical contexts. First, in the interstellar medium, blast waves from supernovae (SNs) or superbubbles radiatively cool after $\sim 10^4$ years and become thin enough for the overstability to act. We will suggest in § V that the overstability could explain why middle-aged and older supernova remnants (SNRs) appear filamentary in optical emission. In that section, we also point out that the overstability acts before gravitational instability can occur in the swept up shell. Thus, the development of the overstability will determine the initial conditions for possible gravitational collapse and star formation in swept up shells of interstellar gas.

Second, blast waves driven into the primordial intergalactic medium have been suggested as a means of biasing galaxy formation (Ostriker & Cowie 1981, Ostriker, Thompson, & Witten 1986, Daly 1987). Again in these scenarios, the overstability acts before gravitational collapse can occur, setting the initial conditions for collapse. Although these ideas have fallen out of favor recently due to the lack of spectral distortion seen by COBE (Levin, Freese, and Spergel 1992), such blast waves formed the original context for the perturbation calculation by Vishniac (1983). Blast waves in the intergalactic medium may still be relevant to the formation of globular clusters.

The forces driving the overstability can be simply described. Consider a thin, decelerating shell driven supersonically into a homogeneous gas by a hot, pressurized interior. Thermal pressure acts on the shell from within, while ram pressure acts across the shock front, as shown in Figure 1. The internal pressure acts normally to the shell surface, since the local sound speed exceeds the expansion speed of the shell. The external ram pressure acts along the shell velocity vector, however, since the shell moves supersonically into the ambient gas. If the surface of the shell is perturbed, the vector sum of the thermal pressure and the ram pressure will yield a resultant force parallel to the shell surface that drives transverse flows from the leading peaks to the trailing valleys. This force concentrates shell material in the valleys, raising their surface density and lowering the surface density of the peaks. Ram pressure deceleration then acts more strongly on the rarefied peaks than on the denser valleys, so that the former peaks end up lagging the former valleys. Now the transverse flows reverse direction, continuing the cycle. Vishniac (1983) showed with his linear analysis that in thin enough shells, the peaks and valleys grow with each cycle.

Grun et al. (1991) have experimentally confirmed the existence of the overstability. They formed blast waves in a test chamber by vaporizing a foil target with a nanosecond pulse from a high energy laser. When the chamber was filled with nitrogen, having $\gamma = 1.4$, the blast wave remained smooth and spherical. However, when the test chamber was filled with xenon, having an effective $\gamma \sim 1.06$ due to radiative cooling, the blast wave wrinkled up.

The linear theory predicts overstability for $\gamma < 1.2$, consistent with the experiment. Grun et al. measured the growth of the overstability, and found the power law time dependence predicted by Vishniac (1983). However, they found that the perturbations ceased to grow at late times.

The nonlinear development of the overstability has not yet been calculated. Numerical calculations of radiatively cooled shells around stellar wind bubbles (Różyczka 1985) and superbubbles (Mac Low, McCray & Norman 1989) have reached high enough density contrasts for the overstability to appear. Both groups found that higher density clumps appeared in the shell, but neither group could confirm that this was due to the dynamical overstability. In both cases an entire quadrant of the shell was modeled with a few $\times 10^5$ cylindrical zones, so that the dynamics of the shell were unresolved. The nonlinear evolution of instabilities in driven blast waves has recently been studied by Chevalier, Blondin & Emmering (1992).

In this work, we use the numerical gas dynamics code ZEUS-2D to compute the nonlinear development of the overstability, as well as to confirm the linear perturbation calculations of the growth rate of the overstability. In § II we describe how we performed our calculations. In § III we compare our calculations to the linear perturbation analysis, while in § IV, we describe the nonlinear development. Finally, we discuss and summarize our results in § V.

2 Technique

To study the overstability, we set up a model of a self-similar blast wave, and then perturb it with an eigenmode of the overstability taken from the linear calculation. In this section we describe these details, and show the accuracy with which we can reproduce the analytic structure of a blast wave.

The gas dynamics code that we use, ZEUS-2D, has been described in detail by Stone & Norman (1992), so we will only give an overview of its algorithm here. It integrates the equations for the dynamics of a perfect gas neglecting viscosity. It uses the Van Leer (1977) formulation for second-order, monotonic advection on an Eulerian grid in Cartesian, cylindrical, or spherical coordinates. Each grid line can be assigned a velocity, allowing the grid to move as desired. Shocks are resolved with a quadratic artificial viscosity, and a small linear artificial viscosity is used to damp oscillations in regions of stagnant flow. The adiabatic index, γ can be set as needed. Many other physics options exist, but do not enter our calculation. This clean version of the code runs at $\sim 3.5 \times 10^5$ zone cy s^{-1} on a Cray Y-MP.

To concentrate zones in the thin, dense shell, we set up a spherical polar grid on an $18°$ wedge centered on the equator. We chose an angle of $18°$ to capture two of the most unstable wavelengths at $\gamma = 1.1$. Radially, the grid extends from a driving piston to 105% of the shock radius r_s. On this grid, the shock slowly moves out from the inner boundary, but never leaves the grid, and is always well resolved. To further optimize the calculation, the active grid only extends ten zones beyond the shock front at each time step.

We want to reproduce the similarity solution for a blast wave as accurately as possible so that we can quantitatively compare our results to the analytic perturbation theory in the small perturbation limit. The usual technique of depositing thermal energy in the central zones and allowing the resulting blast wave to relax to the correct solution leaves sound waves rattling around the interior that tend to mask the initial growth of a small perturbation. Instead, we place a reflecting boundary condition on the inner edge of our grid so that it will act as a piston. We force the piston to follow the path of a parcel of gas that crosses the shock front at the start of our calculation. At each timestep, to find the new position of the piston, we numerically invert the analytic solution for the Lagrangian path of a parcel of gas[1] given by Sedov (1959, 1980, eq. 11.16).

Once the blast wave has been established, we add to it a perturbation consisting of an eigenfunction of the linear overstability with amplitude η and complex eigenvalue $s = s_r + \mathrm{i}s_i$ (Ryu & Vishniac 1987). These eigenfunctions have the form

$$\begin{aligned} \delta\rho(r,\theta) &= \eta\rho_s Y_{lm}(\theta,\phi) d(r/r_s) t^s, & (1)\\ \delta P(r,\theta) &= \eta P_s Y_{lm}(\theta,\phi) p(r/r_s) t^s, & (2)\\ \delta v_r(r,\theta) &= \eta v_s Y_{lm}(\theta,\phi) u(r/r_s) t^s, & (3)\\ \delta v_t(r,\theta) &= \eta v_s \nabla_T Y_{lm}(\theta,\phi) v(r/r_s) t^s, & (4) \end{aligned}$$

where r_s is the shock radius, ρ_s, P_s, and v_s are the post-shock density, pressure, and velocity, respectively, $Y_{lm}(\theta,\phi)$ is a spherical harmonic, $d(r/r_s)$, $p(r/r_s)$, $u(r/r_s)$, and $v(r/r_s)$ are the radial eigenfunctions for density, pressure, radial velocity, and tangential velocity as a function of the normalized radius, and t is the dynamical time (measured from the time at which the explosion occurred). For a quantity Q, with an unperturbed value $Q_0(r)$, we define $\delta Q(r,\theta) = Q(r,\theta) - Q_0(r)$. Both the radial eigenfunctions and the eigenvalue s have complex values, so the perturbations δQ have complex value as well. We take the real part of the perturbations when we perturb the blast wave. The linear calculation shows that the shock radius varies as $\eta r_s Y_{lm}(\theta,\phi)t^s$. To avoid having to perturb the position of the shock front, we chose the perturbation time so that the real part of the radial perturbation would vanish. If we define the oscillation phase $\tau = (s_i/2\pi)\ln(t/t_0)$, our perturbation time is at $\tau = 1/4$. The eigenfunctions and eigenvalues were calculated by Ryu & Vishniac (1987). Dongsu Ryu

[1]The English translation of the ninth edition of Sedov's text (1980) has a typographical error in the definition of his α_7, whose numerator should be $1-\gamma$ rather than $1-\nu$. The translation of the fourth edition (Sedov 1959) has a different error in the same set of equations: the last exponent in the equation for the normalized Lagrangian radius should be $-(\alpha_6+\alpha_7)+2/(2+\nu)$.

kindly supplied us with the code that he used for calculating radial eigenfunctions, allowing us to use an exact eigenfunction to set up the perturbation.

The initial dynamical time t_0 is calculated from the initial shock radius at the beginning of the calculation r_0 (the radius of the inner edge of the grid at the first timestep) by using the similarity solution

$$t_0 = r_0^{5/2}(\rho_0/\xi E)^{1/2}, \tag{5}$$

where E is the energy of the blast, and the similarity constant ξ depends on the adiabatic index γ. We used the two-power approximation of Ostriker & McKee (1989, §IV.B.3) to calculate ξ. In all of the models presented in this paper, we take the energy $E = 250$, the ambient density $\rho_0 = 1$, the sound speed $c_s = 1$, and $\gamma = 1.1$, so that $\xi = 0.2925$. The arbitrary units are defined by these values, so as to keep most quantities of order unity.

We reproduce the similarity solution well. In Figure 2 we show the normalized fractional error in the shock radius as a function of time for several different resolutions (models T1, T2, and T3 from Table 1). As can be seen, the solutions never diverge by more than 0.6%. The dominant jagged noise term actually occurs because we define the shock position to be the zone with the highest density, which only gives the position to the nearest zone. The jaggedness occurs as the analytic shock traverses the zone. Getting the right density jump at the shock forms the most demanding test of a shock solution, especially for the low values of γ and high density ratios that we consider. Because we solve an internal energy equation rather than a total energy equation, we do not automatically get the correct post-shock density. To approach the correct value, we use a large value of the quadratic artificial viscosity, spreading the shock over six zones. This reduces truncation errors to an acceptable value. In Figure 3 we show the peak density as a function of time, along with the exact analytic value. The code does quite well, but it does not reach the analytic value. We discuss possible effects on the overstability in the next section.

3 Linear Regime

In this section we use ZEUS-2D to reproduce the results of the linear perturbation analysis. We place a small perturbation on the blast wave and watch its growth. This allows us to confirm the linear analysis and to check how accurately our code captures the structure of the overstability. We model blast waves with adiabatic index $\gamma = 1.1$ as a compromise between the difficulty of modeling an extremely thin shell and the need to have a thin enough shell for the overstability to occur. The linear analysis shows that the most overstable wavenumber at this value of γ is $l = 40$, with eigenvalue $s = 0.49 + 3.2\,\mathrm{i}$.

We ran two different models in the linear regime. Both had the same initial energy, but the initial radius of model B was ten times larger than model A, resulting in a factor of ~ 300 difference in initial Mach number. The linear analysis was done in the strong shock limit. As we showed in Figure 3, the density behind the lower Mach number shock begins to drop off rather quickly from the strong shock limit, so we expect the higher Mach number model to agree better with the linear analysis. The lower Mach number models, however, are more relevant to typical interstellar conditions.

The radius of the shock is not well-defined in the numerical models (as mentioned in the discussion of Figure 2), making it difficult to directly compare the linear eigenfunctions with our models. Instead, we use radially averaged values of the flow variables as diagnostics. These include the surface density of the blast wave

$$\sigma(\theta_j) = \frac{1}{r_s(\theta_j)^2} \sum_{i=i_0}^{i_s(j)} \rho_{i,j} r_i^2 \Delta r_i, \tag{6}$$

and the mass-weighted tangential velocity

$$\langle v_t \rangle_j = \frac{1}{\sigma r_s^2} \sum_{i=i_0}^{i_s} (v_\theta)_{i,j} \rho_{i,j} r_i^2 \Delta r_i, \tag{7}$$

where i and j are the radial and angular indexes of the grid, i_0 is the innermost radial zone, and we have approximated the transverse velocity along the shell v_t with the velocity along the angular grid lines v_θ. At each angle, θ_j, we define the index of the shock radius $i_s(j)$ to be the index of the outermost zone where $\rho_{i,j} \geq 1.1\rho_0$. The zone size $\Delta r_i = r_{i+1} - r_i$. In our column density figures, we show the perturbations on the blast wave $\delta\sigma/\langle\sigma\rangle$. We define $\delta\sigma(\theta) = \sigma(\theta) - \langle\sigma\rangle$, where $\langle\sigma\rangle = n_\theta^{-1} \sum_j \sigma(\theta_j)$, and n_θ is the number of angular zones. The corresponding quantity for tangential velocity is $\langle v_t \rangle / v_s$.

In Figure 4 we show the two linear models (A and B in Table 1). The grey scale image shows the column density perturbation $\delta\sigma/\langle\sigma\rangle$ as a function of time and angle. As an aid to quantitative understanding, cuts in time and angle are also shown. The oscillation of gas from peaks to valleys and back again appears as an oscillation from positive to negative column density perturbations in this Figure. In Figure 5*(a)* we show the density weighted tangential velocities for the high Mach number model A. The highest velocity flows occur in the middle of the slopes between the peaks and valleys, so the velocity peaks fall a half wavelength away from the density peaks.

To compare our results with the linear analysis, we fit a complex power law Kt^s to the cuts in time using a χ^2 minimization routine (Bevington 1969, p. 237). The best fit exponents s to the cuts in column density are listed in Table 1. (The caption to Figure 5 gives the very similar exponents found by fitting to the velocity rather than the surface density for the models shown in that Figure). We also show the difference between the fit and the calculation $\epsilon = Kt^s - \delta\sigma/\langle\sigma\rangle$. These residuals are so small that the power law fits cannot be distinguished from the numerical results in Figure 4. The linear analysis yields $s = 0.49+3.2$ i, while our high Mach number model yields $s = 0.52+3.1$ i, in close agreement. This power law growth rate is maintained over three orders of magnitude in time, strongly supporting the validity of the linear analysis.

The lower Mach number model B yields a lower growth rate $s = 0.36 + 3.0\,\mathrm{i}$. The linear theory qualitatively predicts this difference as being caused by the drop off in the peak density in the shell (see Figure 3). Such a drop would occur if the adiabatic index γ were raised. When γ is raised, the growth rate of the overstability drops (Ryu & Vishniac 1987). The overstability is clearly quite sensitive to the density jump across the shock.

4 Nonlinear Regime

Now we turn to the nonlinear development of the overstability. We have shown that the linear theory gives a very good description of the initial growth of the overstability. Therefore our strategy here is to start with larger amplitude perturbations that still take the form of the linear eigenfunctions, but move into the nonlinear regime quickly or immediately. We again ran models at both medium and high initial Mach numbers. We also ran one model at twice the resolution of the others to show that the nonlinear effects we observe are resolved.

We find that weak transverse shocks form as large mass flows collide in the valleys. The overstability saturates and ceases to grow when the tangential velocities along the shell exceed the post-shock sound speed. The oscillation rate s_i remains virtually unchanged, however. In Figure 6 we show the behavior of the surface density as a function of angle and time. Fitting power laws to the two models, we find $s = 0.14 + 3.2$ i for the higher amplitude model F, and $s = 0.33 + 3.1$ i for the moderate amplitude model C. In Figure 7 we show the approach to saturation and the behavior of the fully saturated overstability. (Note that we show density as a function of radius and angle, rather than real distance. As time passes, the aspect ratio of the zones actually changes, though we keep it constant in this figure.)

We plot the maximum transverse velocity and compare it in Figure 8 to the post-shock sound speed $c_p \approx (2\gamma[\gamma - 1])^{1/2}(\gamma + 1)^{-1} v_s$. Transverse shocks first form when the gas flowing into the valleys from both sides collides supersonically, or when transverse velocities exceed Mach 0.5. At this point the solution begins to diverge from the linear one. Lumps form behind the transverse shocks, effectively adding power to smaller wavelengths of the overstability. As expected, these smaller wavelengths grow more slowly than the wavelength of the initial perturbation, which we chose to be the fastest growing wavelength. After transverse shocks form, perturbation growth continues, however, until the fastest transverse flows are supersonic.

At saturation, $\sim 30\%$ variations in column density occur in the shell as shown in Figures 6. As the shock oscillates back and forth $\sim 20\%$ variations in the shock velocity occur. The ripples in our model, as seen in Figure 7, are quite small, with a maximum $\Delta r/r_s \approx 0.02$. The shock front always remains close to circular, with a maximum angle of about $12°$ from the "horizontal".

To show that the overstability really does saturate, and that we have not just found the limits of our numerical resolution, we reran the low Mach number model D at twice the resolution (model E). In Figure 9 we show the development of the two models, as well as the rather small residuals left from subtracting one from the other along a time sample. From this figure we conclude that our 128 zone models resolve the development of the overstability well.

Our results confirm and begin to explain the laboratory measurements of Grun et al. (1991). They found that growth of the overstability stopped at late times. They were uncertain whether they were observing a true saturation of the overstability or some artifact of the experiment. From the analytic theory and the radii and times they quote, the blast they produced was still traveling at over Mach 100 when the saturation occurred at a time of ~ 1000 ns. We believe that they observed overstability saturation as weak transverse shocks formed. Further measurements may be able to show these shocks.

We have only studied the two-dimensional case of azimuthal symmetry, with perturbations only in the radial and polar directions. We can speculate on the full three-dimensional development of the overstability. At low amplitudes, linear combinations of spherical harmonics $Y_{lm}(\theta, \phi)$ for $m \neq 0$ probably still describe the eigenfunctions of the overstability. As the perturbations grow, differences will appear from the two-dimensional case. The focussing of mass into point valleys will be more effective in three dimensions, although linear valleys will still behave as they did in two dimensions. This will produce occasional large spikes that grow larger than indicated by our computation. Such spikes can be seen in the experiments of Grun et al. (1991).

5 Discussion

5.1 Shell Fragmentation and Collapse

Perhaps the first impression we get from our models is that the overstability does not tear the shell into fragments, or even strongly disrupt the shock, unlike the apocalyptic picture painted by Vishniac (1983, pp. 164-165). To quantify this, we found the peak density along each radial grid line, and show in Figure 10 the ratio between the highest and the lowest peak densities in our most nonlinear model. Although density contrasts of a factor of two do occur along the shock front, coherent fragments do not persist as the shock oscillates, as shown in the second sequence of Figure 7.

The overstability ceases to grow when the Mach number drops too low. This turns out to be a fairly stringent limit on the growth of perturbations. The overstability grows as t^s, where $s = s_r + is_i$. Thus, perturbation size grows as t^{s_r}; where $s_r \approx 1/2$ at the most overstable wavelengths according to the linear analysis. The shock velocity, however, drops as t^{-a}, where $a = 3/5$ for point explosions and $a = 2/5$ for bubbles. If the overstability sets in at a velocity v_0, and ceases growth at a velocity v_{min}, the maximum amplification is $(v_{min}/v_0)^{-s_r/a}$, so the maximum growth is nearly linearly proportional to the velocity range covered by the shock. For a SNR, the overstability sets in when the decelerating shell cools at a shock velocity of $\sim$150–200 km s^{-1}. If the SNR is expanding into 10^4 K gas having a sound speed of 10 km s^{-1}, perturbations will only be amplified by a factor of 30. To produce strong effects in this case, the initial perturbations must be several percent in amplitude. However, if the initial Mach number is higher, as in dense clouds, more growth can occur.

Now let us consider how the overstability will affect the time scale of gravitational instability t_G. Elmegreen (1989) calculated the gravitational collapse of isothermal shocked layers in the linear regime. He found that strong transverse flows only permit the monotonic collapse of masses larger than a threshold mass having an escape speed of about half the shock velocity. The collapse time on this large scale is relatively long, $t_G \sim 0.25(G\rho_0)^{-1/2}$, and is insensitive to the Mach number, M. Vishniac (1983) omitted the transverse flows in his calculation of the gravitational collapse problem. He derived a smaller threshold mass, with an escape speed of the post-shock sound speed, and a correspondingly shorter collapse time, $t_G = M^{-1/2}(G\rho_0)^{-1/2}$. Here we have shown that the transverse flows behind the shock do not approach the shock velocity, but only reach the post-shock sound speed, so that the low mass threshold and short collapse time derived by Vishniac (1983) apply.

5.2 Filamentary Structure in Supernova Remnants

Middle-aged and older SNRs with radiative shocks such as Vela and the Cygnus loop have extensive filamentary structure at optical wavelengths. The most convincing explanation of the nature of the filaments was advanced by Hester (1987), who argued that rippled sheets viewed at various angles could reproduce the observed structure. The observed emission lines arise in the thin recombination zone immediately behind the shock front, creating the thin sheet needed by this explanation.

The source of the ripples has been thought to be "weather"; that is, small, random variations in the pre-shock gas. Several observations suggest a different answer. First, filaments are observed on a wide variety of scales, even though H I observations do not reveal variations on those scales in the ambient gas (e.g. Payne, Salpeter & Terzian 1983). The rippled sheet model reduced this problem from the small cloud model, but it still remains. Second, filamentary structure in the Cygnus Loop becomes more complex as shocks becomes more radiative (Hester 1987, Hester, Raymond, & Danielson 1986). Third, filamentary structure is much more pronounced in SNRs than in wind-blown bubbles, even though both have radiative shocks. Finally, filaments in the Cygnus Loop and in Vela show blister structures in velocity space (Greidanus & Strom 1988; Meaburn, Hartquist & Dyson 1988).

We would like to suggest that we are observing the dynamical overstability at work wrinkling up radiative blast waves. In this paper we have shown that the end result of the development of the overstability is a shell that remains wrinkled indefinitely (Figs. 6*(b)* and 7). The experimental results of Grun et al. (1991) easily reproduce the wrinkled sheet geometries of the analog models that Hester (1987) used to explain filaments in the Cygnus Loop. The overstability acts more strongly when the density contrast is higher, producing more complex ripples, as observed. The overstability is stronger in SNRs than in bubbles (Vishniac & Ryu 1987, 1988) because SNRs decelerate more rapidly than bubbles ($v_s \propto t^{-3/5}$ instead of $v_s \propto t^{-2/5}$). Finally, blister structures can be seen in Figure 7 as the density concentrations move forward and break apart.

Our suggestion explains the empirical observation of filamentary structure in SNRs with no reference to unobserved properties of the ambient gas. Any decelerating, radiative blast wave will be rippled or wrinkled, even in a smooth ambient medium. Optical emission from a thin layer on the surface of such a blast wave will show filamentary structure along any caustic tangent to the line of sight. Note that we do not observe the dense shell, but only a thin layer on its surface. The typical wavelength of the ripples will be the most overstable wavelength, a few percent of the shell radius. This is consistent with observations of the Vela nebula (Meaburn et al. 1988).

To make quantitative comparisons between the shapes and velocity structures of SNRs and overstable blast waves will require three-dimensional models. The experimental results indicate that longer wavelengths can be excited to higher amplitudes in three dimensions than they can be in two. The other question is how true radiative cooling will affect the dynamics of the shell. We speculate that the shell will behave as if it had the effective adiabatic index that would be indicated by the density jump across the radiative shock. We think future models can and should include both these effects.

5.3 Summary

We now summarize our main results.

1. We have quantitatively confirmed the overstability of thin blast waves predicted by the linear analyses of Vishniac (1983) and Vishniac & Ryu (1987).
2. The growth of the overstability in adiabatic shocks saturates when weak transverse shocks form in the shell. Transverse velocities in the shell approach the post-shock sound speed.
3. Our calculations confirm and begin to explain the experimental results of Grun et al. (1991) who saw growth of the overstability followed by saturation in the nonlinear regime.
4. We suggest that this overstability explains the filamentary structure of middle-aged and older SNRs observed at optical wavelengths.
5. The overstability does not fragment the shell, but does produce density enhancements of more than a factor of two and transonic flows in the shell.

We thank Dick McCray for suggesting this project and participating in its early stages. We also thank Dongsu Ryu for giving us his code for generating eigenfunctions, Ethan Vishniac, Bruce Elmegreen, Mark Voit, and David Weinberg for helpful discussions, Jim Stone for his work on ZEUS-2D, and Kevin Zahnle for a close reading. M.-M.M.L. thanks the NASA Astrophysical Theory Center for Star Formation Studies for support, and the NCSA and Dartmouth College for hospitality during work on this paper. The computations discussed in this paper were performed at the National Center for Supercomputing Applications.

Table 1: Models

Model	η^a	$\delta\rho/\rho_s{}^b$	$r_0{}^c$	$n_r{}^d$	$n_\theta{}^d$	$M_0{}^e$	s^f
A	5(-4)	0.0070	0.01	128	256	3420	0.52+3.1i
B	5(-4)	0.0070	0.1	128	256	108	0.36+3.0i
C	5(-3)	0.070	0.01	128	256	3420	0.33+3.1i
D	2(-2)	0.28	0.1	128	256	108	0.12+2.9i
E	2(-2)	0.28	0.1	256	512	108	0.11+3.0i
F	3(-2)	0.42	0.01	128	256	3420	0.14+3.2i
T1	0	0	0.1	64	2	108	-
T2	0	0	0.1	128	2	108	-
T3	0	0	0.1	256	2	108	-
T4	0	0	0.01	128	2	3420	-

[a] Amplitude of initial perturbation.

[b] Initial density contrast.

[c] Initial radius of shock.

[d] Number of zones in radial and angular directions.

[e] Initial Mach number of blast wave

[f] Complex power law of overstability. For a strong shock with $\gamma = 1.1$ and wavenumber $l = 40$, the linear theory predicts $s = 0.49 + 3.2$ i.

List of References

Bertschinger, E. 1986, ApJ, 304, 154

Bevington, P. R. 1969, Data Reduction and Error Analysis for the Physical Sciences (New York: McGraw Hill)

Chevalier, R. A., Blondin, J. M., & Emmering, R. T. 1992, ApJ, 392, 118

Daly, R. A. 1987, ApJ, 322, 20

Elmegreen, B. G. 1989, ApJ, 340, 786

Erpenbeck, J. J. 1962, Phys. Fluids, 5, 1181

Greidanus, H. & Strom, R. G. 1988, in IAU Colloq. 101, Supernova Remnants and the Interstellar Medium, eds. R. S. Roger & T. L. Landecker (Cambridge: Cambridge Univ. Pr.), p. 443

Grun, J., Stamper, J., Manka, C., Resnick, J., Burris, R., Crawford, J., & Ripin, B. H. 1991, Phys. Rev. Lett., 66, 2738

Hester, J. J., Raymond, J. C., & Danielson, G. E. 1986, ApJ, 303, L17

Hester, J. J. 1987, ApJ, 314, 187

Levin, J. J., Freese, K., & Spergel, D. N. 1992, ApJ 389, 464

Mac Low, M.-M., McCray, R. A., & Norman, M. L. 1989, ApJ, 337, 141

Meaburn, J., Hartquist, T. W., & Dyson, J. E. 1988, MNRAS, 230, 243

Ostriker, J. P., & Cowie, L., L. 1981, ApJ, 243, L127

Ostriker, J. P., Thompson, C., & Witten, E. 1986, Phys. Lett., B180, 231

Ostriker, J. P. & McKee, C. F. 1988, Rev. Mod. Phys., 60, 1

Payne, H. E., Salpeter, E. E., & Terzian, Y. 1983, ApJ, 272, 540

Różyczka, M. 1985, A&A 143, 59

Ryu, D. & Vishniac, E. T. 1987, ApJ, 313, 820

———. 1988, ApJ, 331, 350

Sedov, L. I. 1980, Similarity and Dimensional Methods in Mechanics, Ninth Ed., trans. V. I. Kisin (Moscow: Mir Publishers)

————. 1959, Similarity and Dimensional Methods in Mechanics, Fourth Ed., trans. M. Friedman (New York: Academic Press)

Shull, J. M. 1988, in IAU Colloq. 101, Supernova Remnants and the Interstellar Medium, eds. R. S. Roger & T. L. Landecker (Cambridge: Cambridge Univ. Pr.), p. 161

Stone, J. M. & Norman, M. L. 1992, ApJSupp, 80, 753

Van Leer, B. 1977, J. Comp. Phys., 23, 276

Vishniac, E. T. 1983, ApJ, 274, 152

List of Figures

Figure 4: Comparison of computation with linear perturbation theory. Column density perturbation, $\delta\tilde{\sigma} = \delta\sigma / \langle\sigma\rangle$ is shown as a function of time and angle for models *a)* A and *b)* B, with high and moderate initial Mach numbers. Cuts are shown at the final time and at an angle of $\pi/2$. The residual is shown from the best fit power law $\epsilon = \delta\tilde{\sigma} - K(t/t_0)^s$, with s given in Table 1. The power law fit cannot be distinguished from the numerical result for these models.

Figure 5: Density weighted transverse velocity $\langle v_\theta \rangle$, scaled by the shock velocity for *a)* linear model A and *b)* nonlinear model F. The best fit power laws are shown as thin lines; they are $s = 0.48 + 3.0$ i and $s = 0.085 + 3.1$ i, similar to the values derived for these models from the column density (Table 1). Everything else is similar to Figure 4.

Figure 6: *(a)* Transition to nonlinearity (model C) and *(b)* full nonlinear evolution (model F), shown in same way as Figure 4. The best fit power laws are shown as thin lines in the time cut.

Figure 7: Greyscale images of density field for model F. The time t/t_0 and the phase $(s_i/2\pi)\ln(t/t_0)$ are shown in each panel. The palette has been rotated to highlight density peaks as indicated in the scale bar. The first sequence shows the initial development of the overstability following a strong perturbation at phase 0.25, and the second sequence shows half a cycle of the fully nonlinear overstability. Note that the spherical polar grid is displayed on a rectangle (ignoring the actual decrease of zone aspect ratio with time), and that the grid moves to follow the shock.

Figure 8: Maximum transverse velocity v_θ on the grid for the nonlinear model F (solid line) compared to the linear model A, scaled to the same perturbation size (dotted line), showing the damping that occurs when the transverse velocities approach the post-shock sound speed (thin line). Velocities are scaled by the shock speed v_s.

Figure 9: Resolution study at low Mach number, similar to Figure 4, except that medium resolution model D (top, dotted lines) and high resolution model E (bottom, solid lines) are shown. The difference between the two time cuts is shown as the residual ϵ.

Figure 10: Maximum and minimum post-shock density for model F (solid lines), along with the post-shock density for the unperturbed model T4 (dashed line). The ratio of the maximum to the minimum densities is also shown as the density contrast.